

Redtail Parrotfish

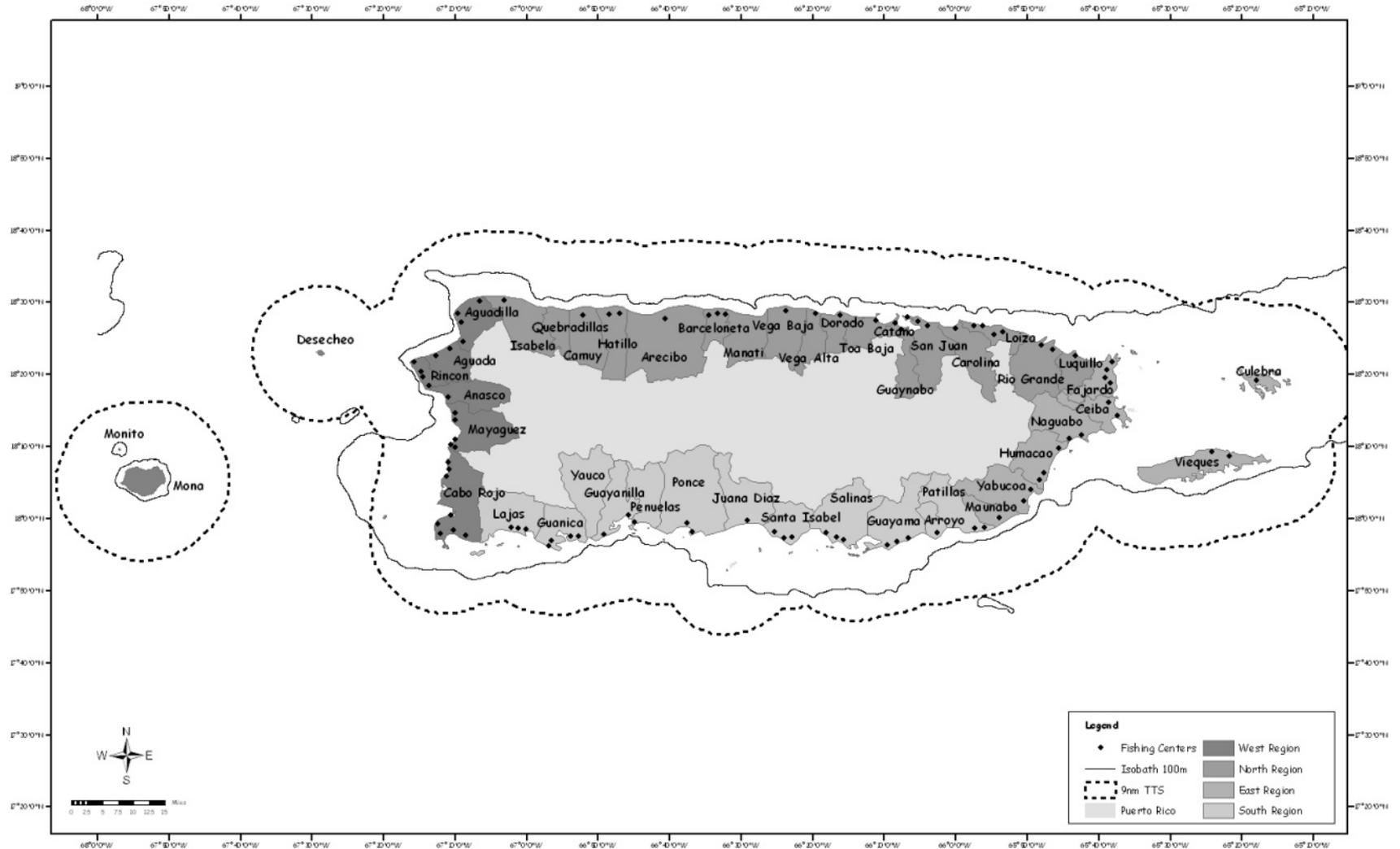


Sparisoma chrysopterygium, redbtail parrotfish. Photo credit J.E. Randall (1997).

**NOAA
FISHERIES
SERVICE**

Todd Gedamke, Nancie Cummings,
Meaghan Bryan and Kevin
McCarthy (SEFSC)

Puerto Rico Commercial Fishing Reporting Zones

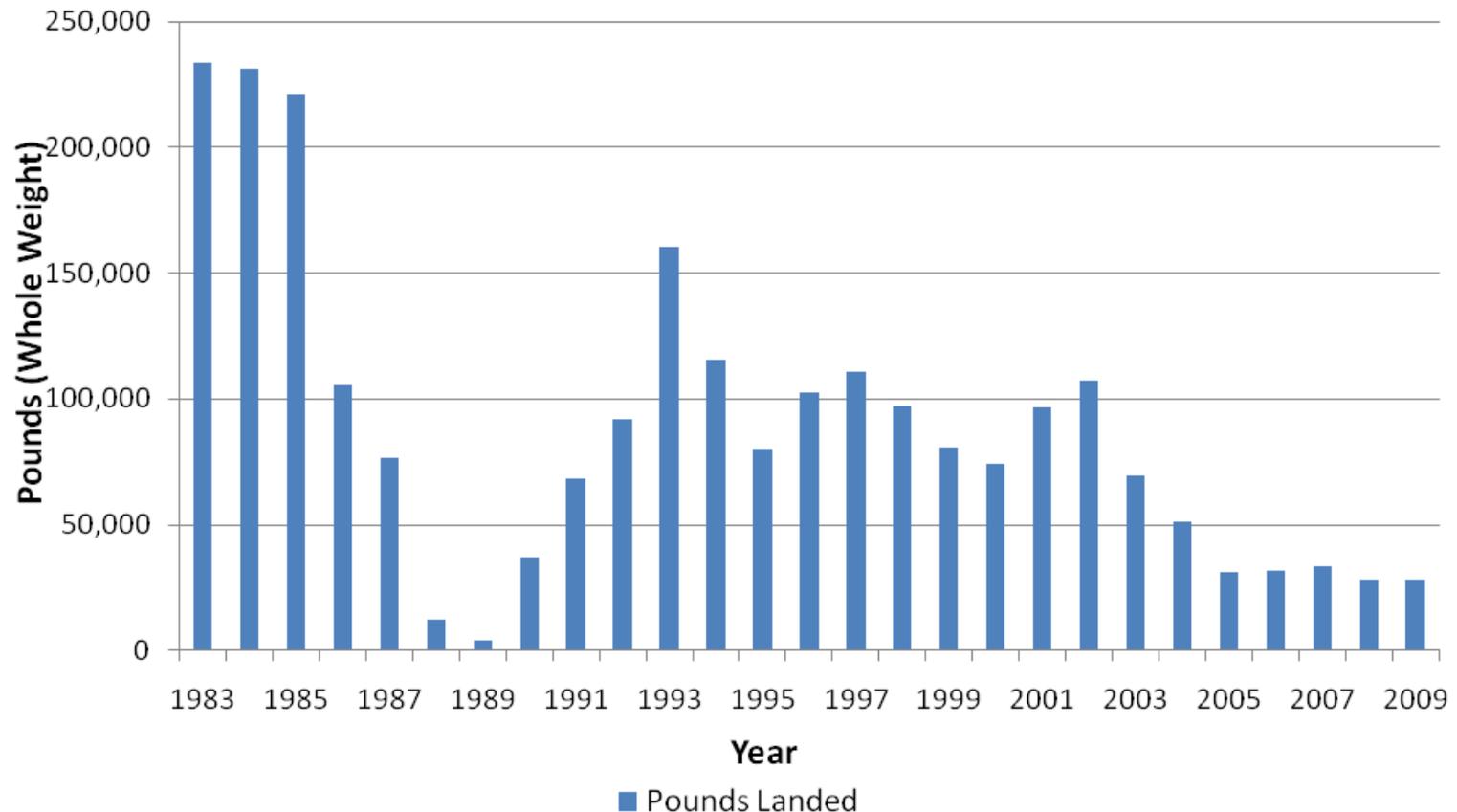


Puerto Rico Landings and Abundance Trends for Parrotfish Species

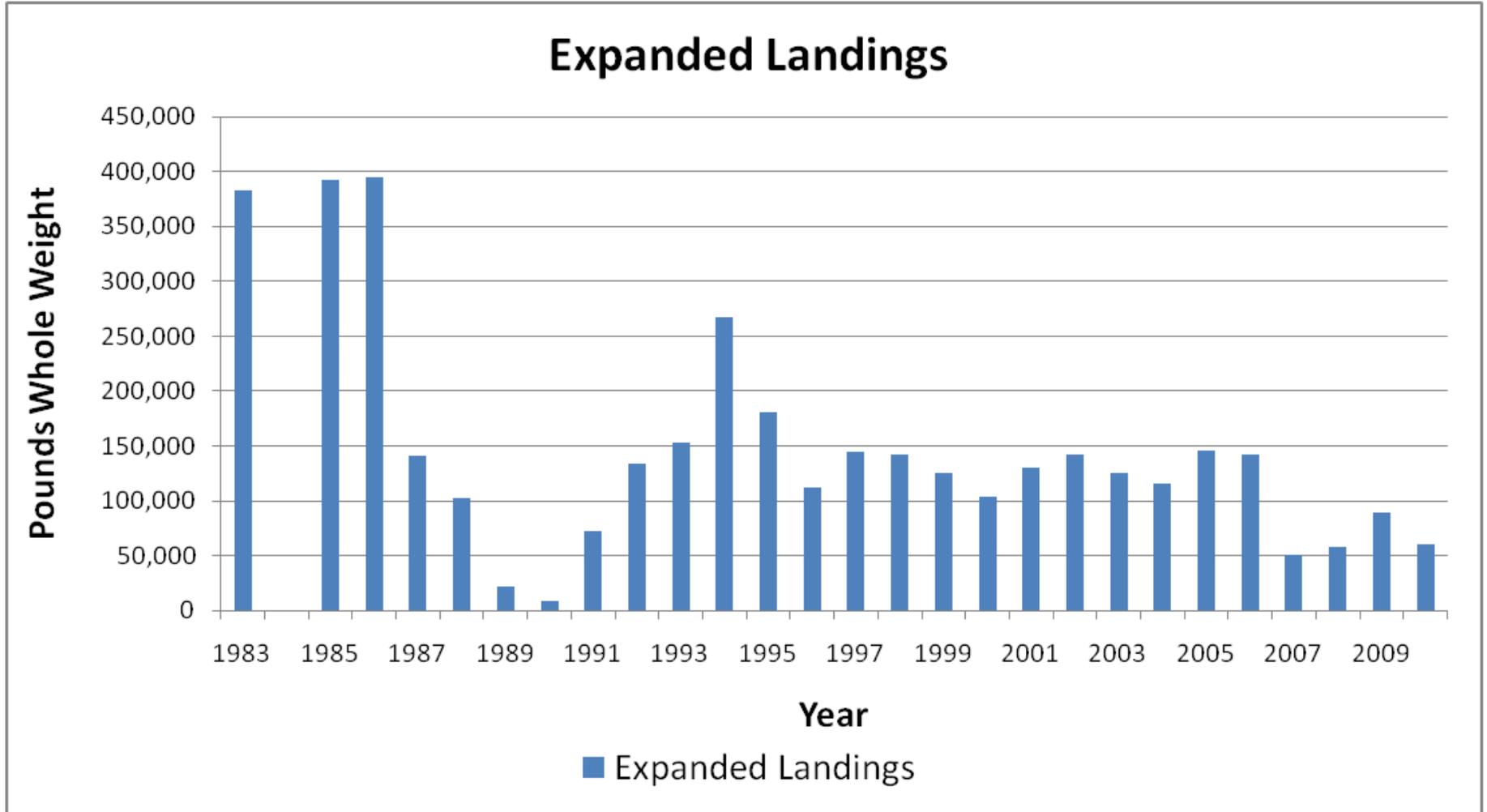
- Landings trends could not be quantified for the Puerto Rico commercial fisheries by individual species as species level data have not been recorded to date by fishers.
- Landings trends profiles were constructed for aggregate parrotfish family group from Puerto Rican commercial landings

Puerto Rico Parrotfish Family Reported Commercial Landings

Reported Commercial Pounds of Parrotfishes Landed in Puerto Rico



Puerto Rico Parrotfish Family Expanded Landings



Index Development Considerations

Species	Gear				
	Handline	Fishpots	Gillnet	Trammel Net	Dive
Silk Snapper (with vermilion snapper, blackfin snapper, and black snapper)	Start Year = 1983+	Start Year = 1983+			
	Gear = 104 + 112 + 113 + 105 Fishing Centers = 01 + 02 + 03 + 05 + 06 + 12 + 13 + 15 + 16 + 18 + 20 + 21 + 22 + 25 + 28 + 29 + 32 + 33 + 35 + 36 + 37 + 38 + 39 + 40 + 41 + 42	Gear = 101 Fishing Centers = 01 + 05 + 06 + 08 + 09 + 10 + 12 + 13 + 14 + 15 + 16 + 18 + 20 + 22 + 23 + 25 + 28 + 32 + 36 + 37 + 38 + 39 + 40 + 41 + 42			
Queen Snapper (with cardinal snapper)	Start Year = 1987+				
	Gear = 104 + 105 Fishing Centers = 01 + 05 + 06 + 12 + 13 + 15 + 16 + 28 + 32 + 35 + 36 + 37 + 38 + 39 + 40 + 41 + 42				
Parrotfish		Start Year = 1983+	Start Year = 1988+	Start Year = 1988+	Start Year = 1997+
		Gear = 101 Fishing Centers = 18 + 19 + 20 + 21 + 22 + 23 + 24 + 25 + 27 + 28 + 29 + 31 + 36 + 37	Gear = 103 Fishing Centers = 23 + 27 + 35 + 36 + 37	Gear = 118 Fishing Centers = 23 + 27 + 35 + 36 + 37	Gear = 110 + 114 + 115 + 116 + 119 Fishing Centers = 14 + 18 + 19 + 20 + 21 + 24 + 25 + 27 + 33 + 34 + 35 + 36 + 37 + 38 + 40

Index Considerations

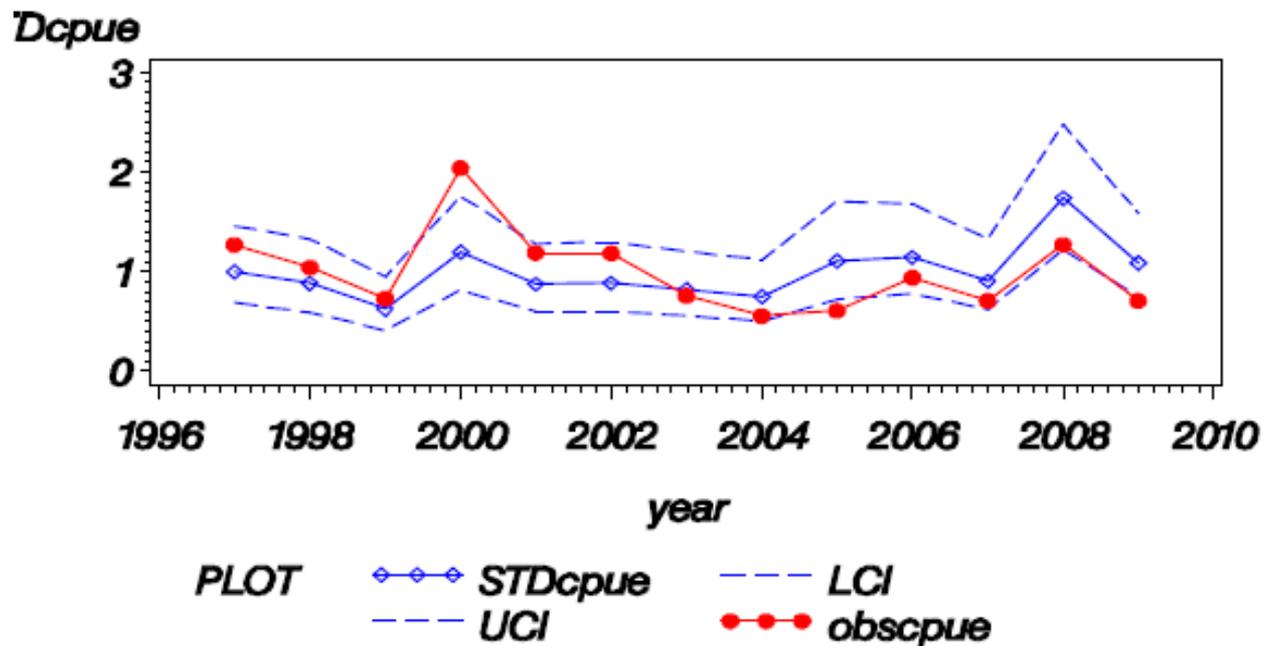
- Puerto Rico fishery platform group reviewed DW-03 (synopsis of fishery)
- Parrotfish were predominately reported from fish pots, gillnets, trammel nets and dive gear
- CPUE abundance set construction restricted to the above four gears
- Delta lognormal fits made to each independent data series

Puerto Rico Parrotfish Family Indices

- Redtail parrotfish abundance trends could not be developed as species specific landings data do not exist for US VI parrotfish category landings
- Some parrotfish family CPUE indices were developed for Puerto Rico
 - Dive
 - Trammel Net and Gillnet
 - Fish Pots

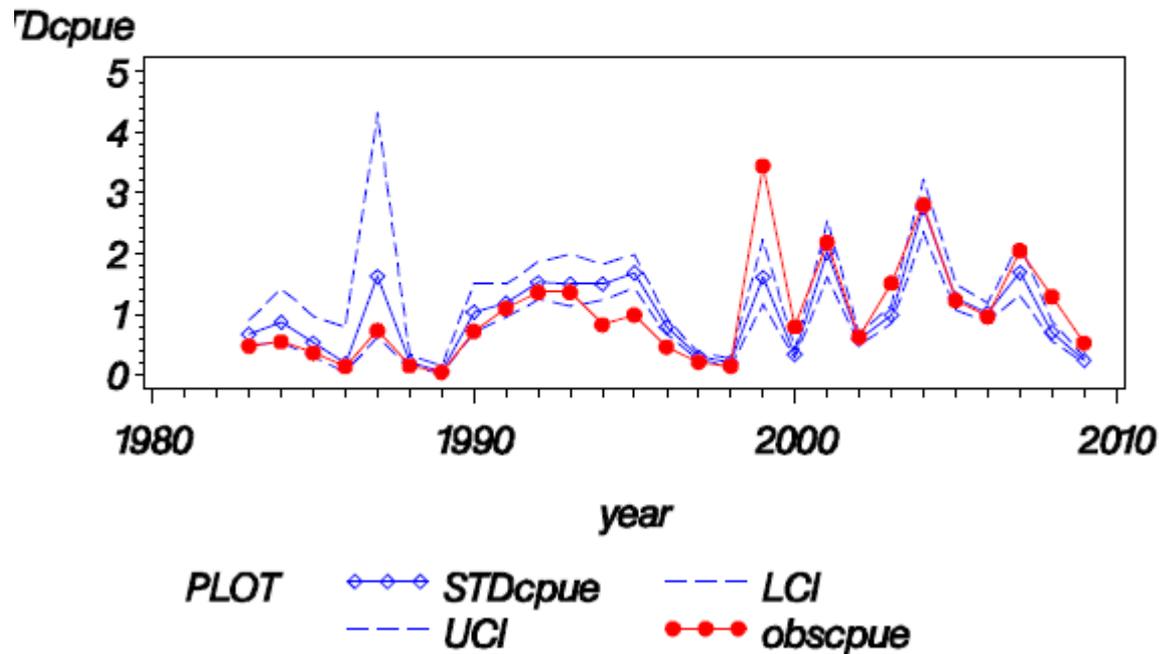
Puerto Rico Parrotfish Family Standardized Dive CPUE

*Catch for Puerto Rico Parrotfish Dive
Observed and Standardized CPUE (95% CI)*



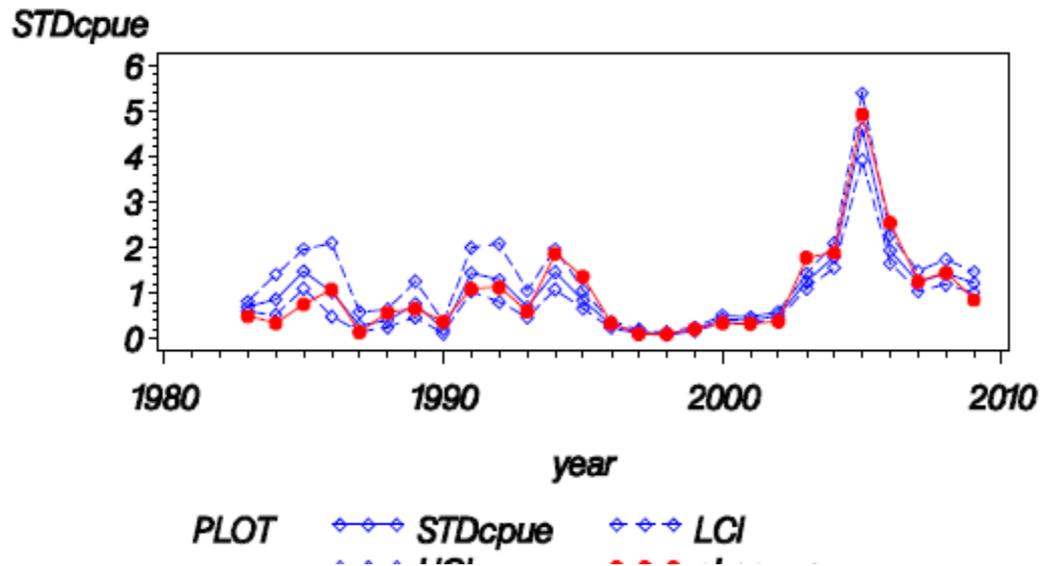
Puerto Rico Parrotfish Family Standardized Trammel Net and Gillnet CPUE

*Catch for Puerto Rico Parrotfish Trammel Net and Gillnet
Observed and Standardized CPUE (95% CI)*



Puerto Rico Parrotfish Family Standardized Fish Pot CPUE

Catch for Puerto Rico Parrotfish Fishpots
Observed and Standardized CPUE (95% CI)

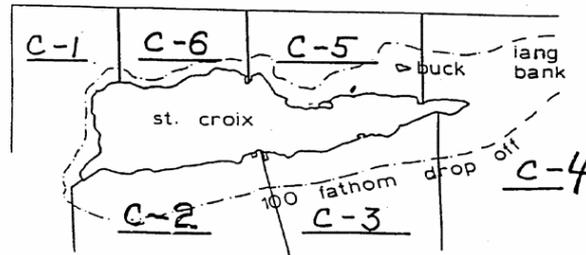
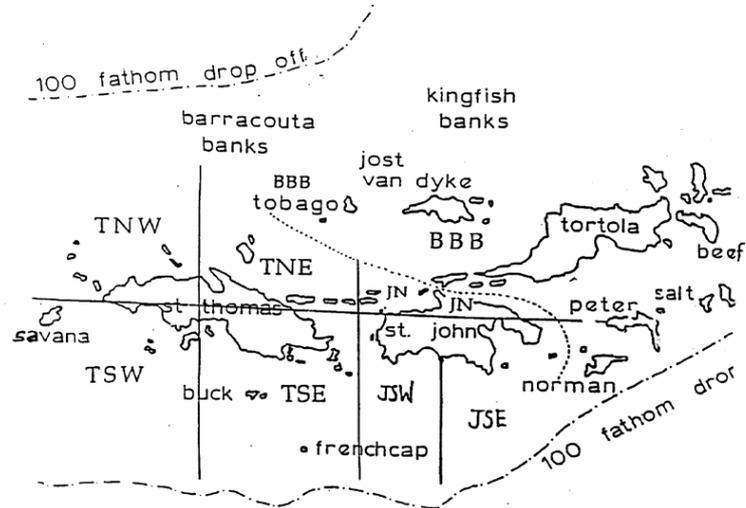


Effort Trends Puerto Rico

- Reporting voluntary until 2006 in Puerto Rico
- Fishery reporting rates have high spatial and temporal variability.
- Expansion factors average $\sim 60\%$ over the entire time series
- Trip Tickets represent multiple years in many cases prior to 2006

US Virgin Islands Fishery Reporting Zones

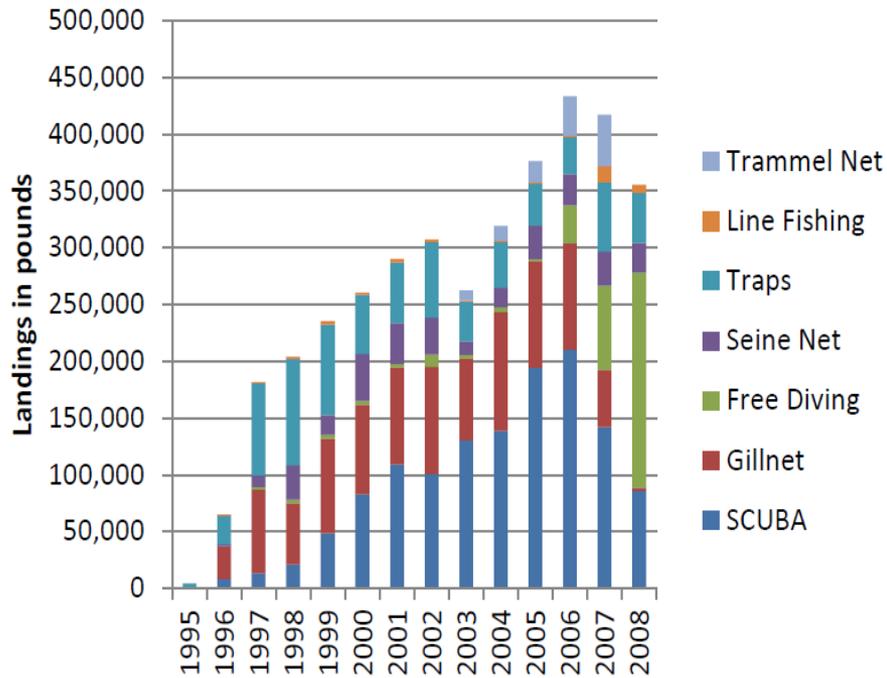
Catch Report Forms, revised
 for Commercial Fisheries of the U.S. Virgin Islands
 July 1999 - June 2000



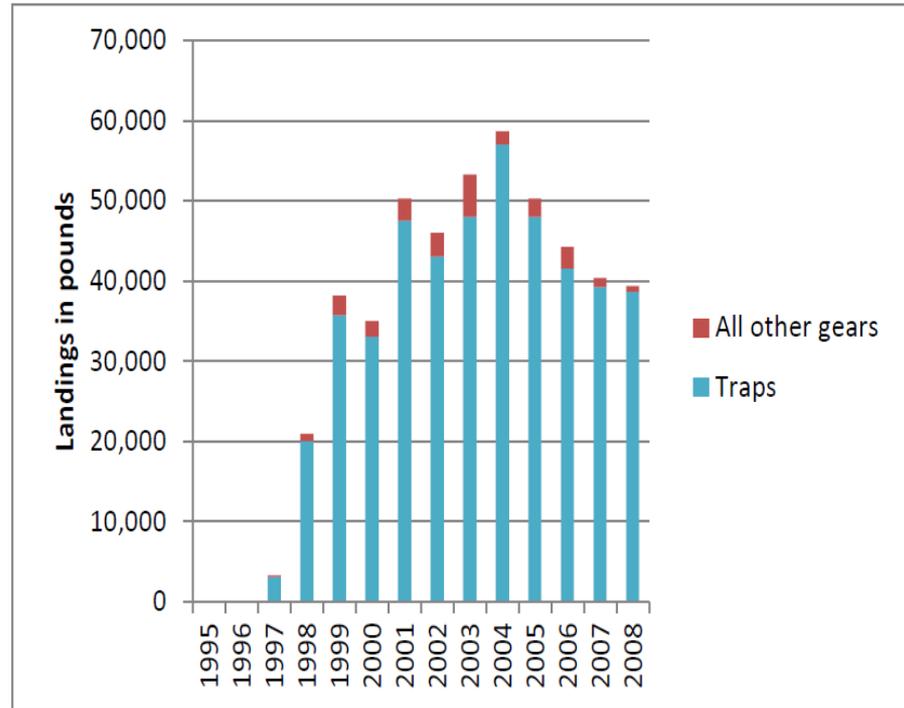
US Virgin Islands Landings and Abundance Trends for Parrotfish Species

- Landings trends could not be quantified for the US Virgin Islands commercial fisheries by species as species level data are not available.
- Landings trends profiles were constructed for aggregate parrotfish family group from US VI reported commercial landings.

US Virgin Islands Parrotfish Family Reported Landings



St. Croix



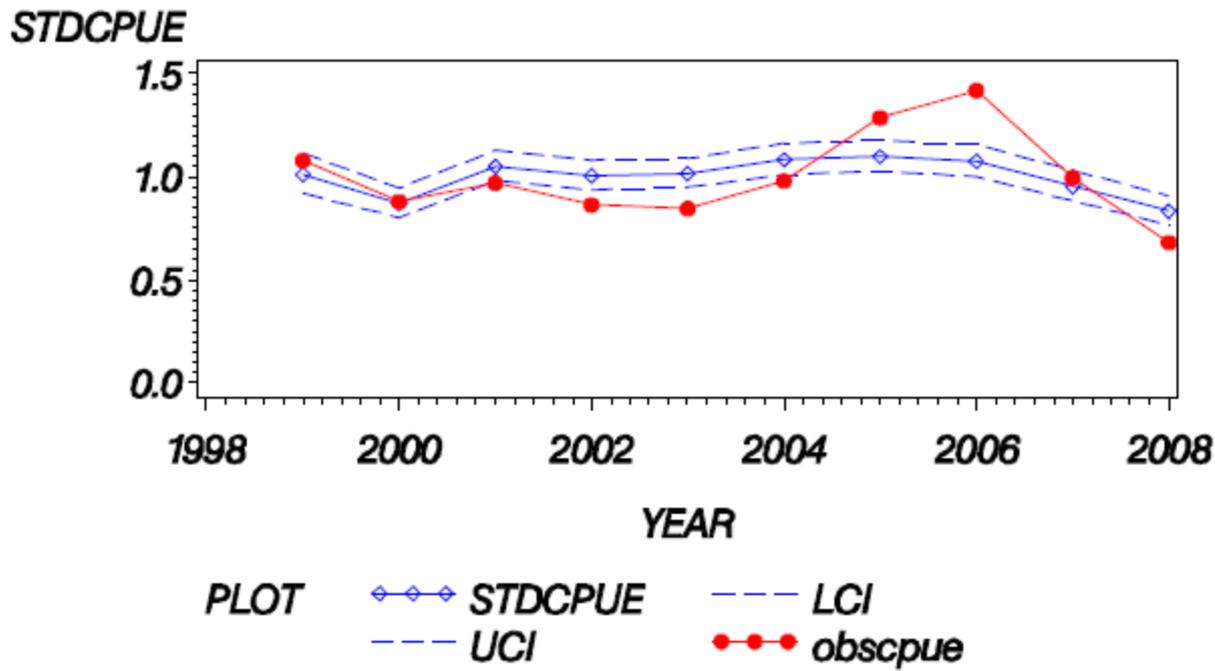
St. Thomas – St. John

USVI Parrotfish Family Abundance Indices

- Redtail parrotfish abundance trends could not be developed as species specific landings data do not exist for US VI parrotfish category landings
- Some parrotfish family CPUE indices were developed for US Virgin Islands
 - Traps (St. Thomas-St. John, St. Croix)
 - Scuba (St. Croix)
 - Gillnet (St. Croix)

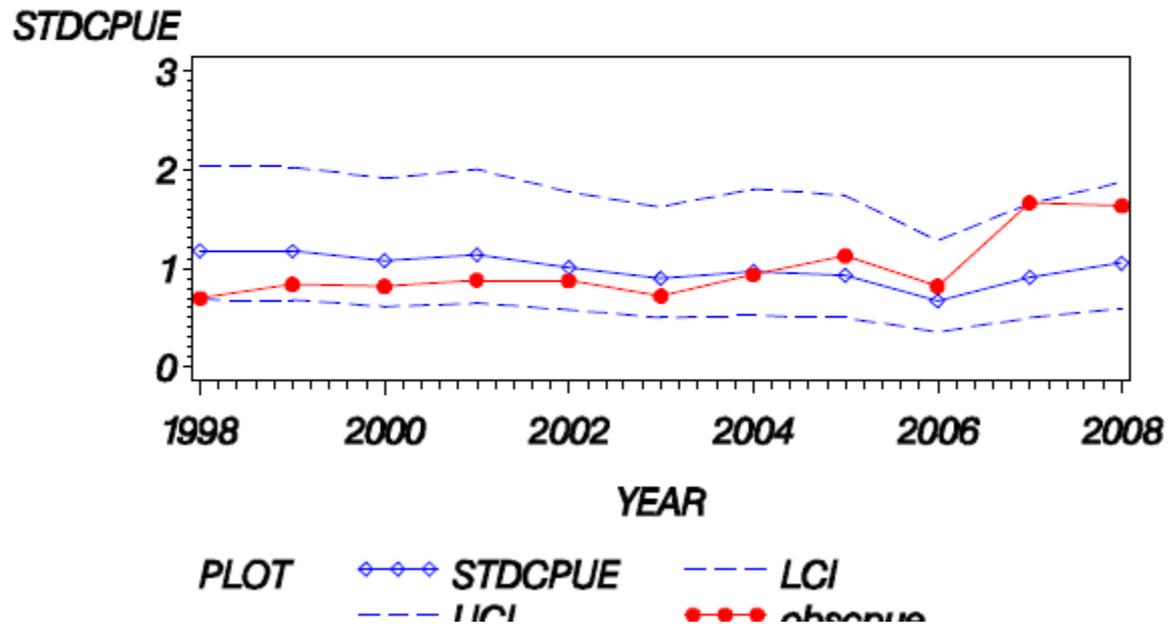
St. Thomas St. John Parrotfish Family Standardized Trap CPUE

*STSJ PARROTFISH TRAP DATA 1999–2008
Observed and Standardized CPUE (95% CI)*



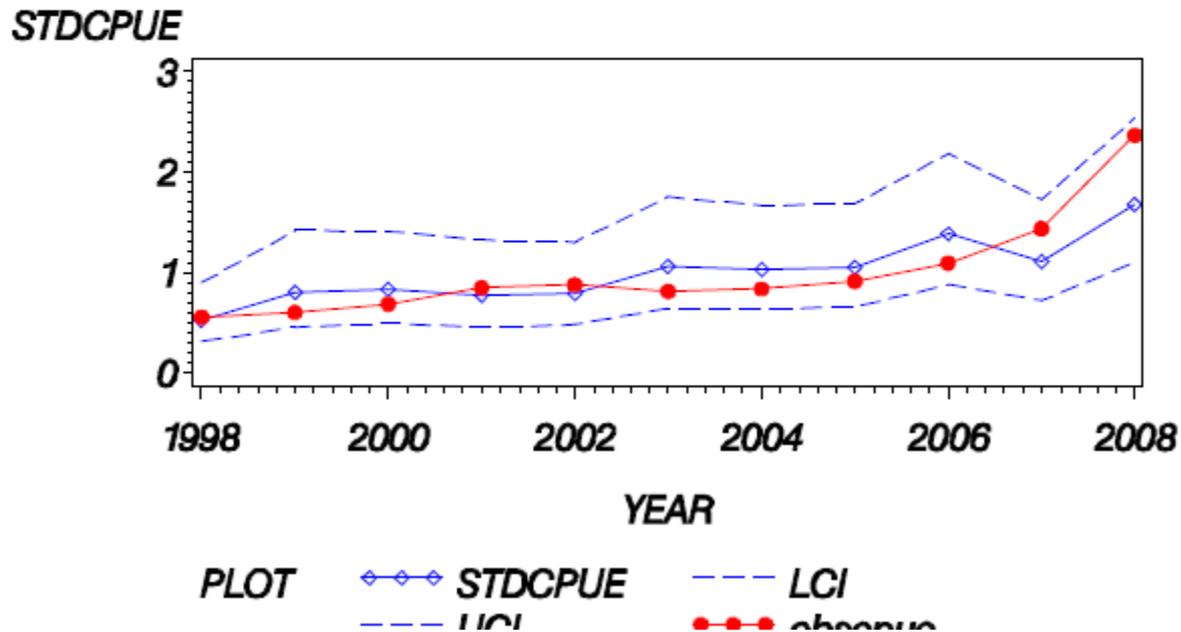
St. Croix Parrotfish Family Standardized Trap CPUE

STX PARROTFISH TRAP DATA 1998–2008
Observed and Standardized CPUE (95% CI)



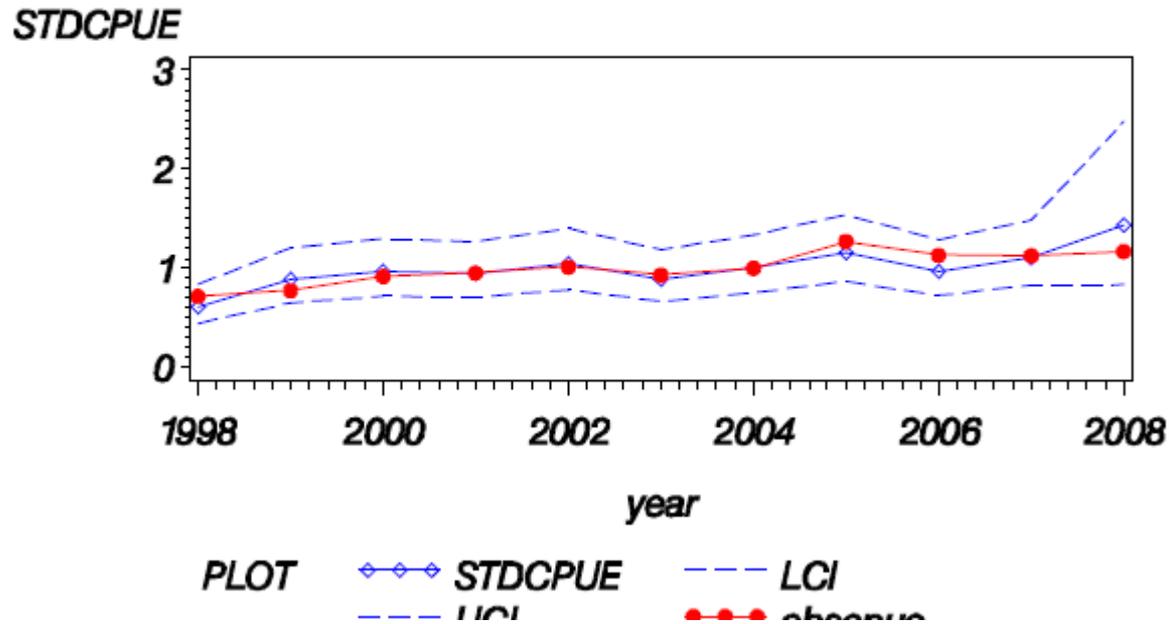
St. Croix Parrotfish Family Standardized Scuba CPUE

STX PARROTFISH SCUBA DATA 1998–2008
Observed and Standardized CPUE (95% CI)

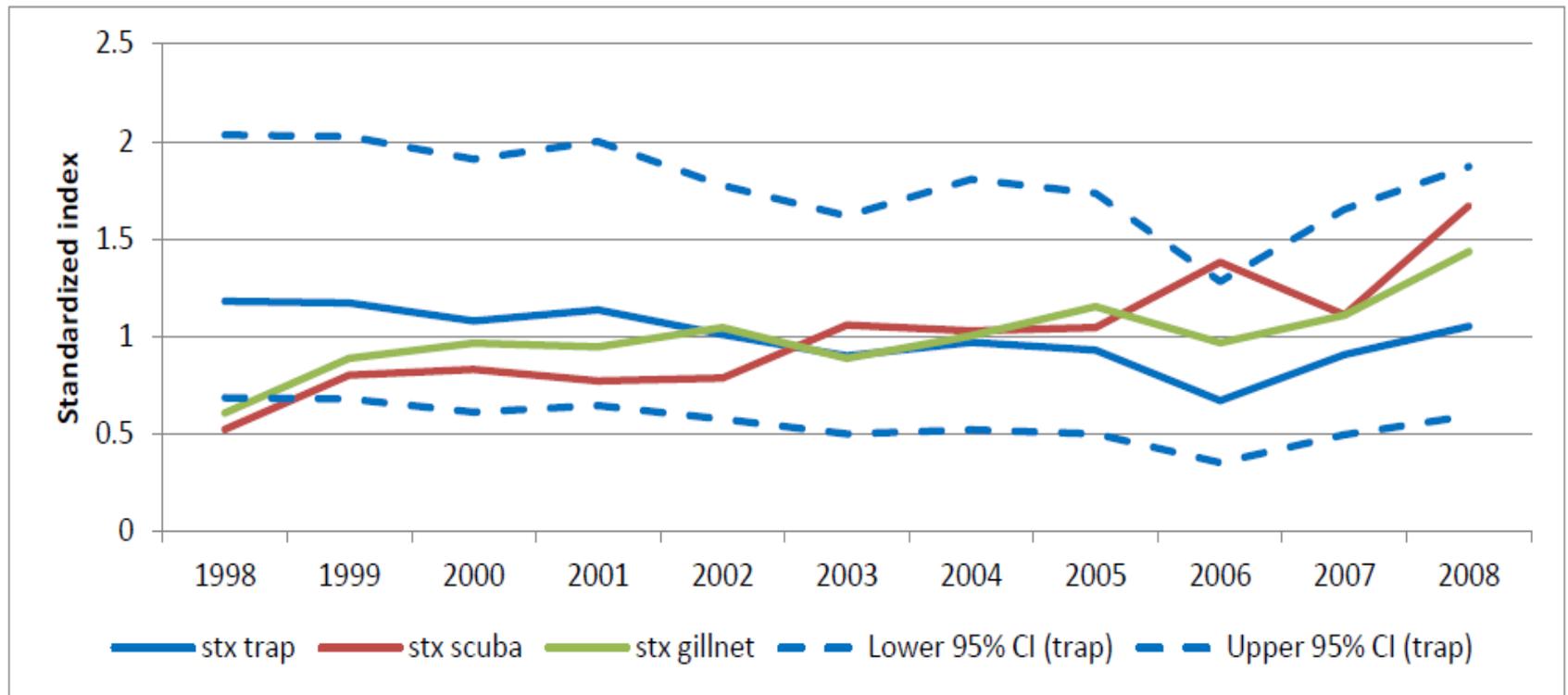


St. Croix Parrotfish Family Standardized Gillnet CPUE

STX PARROTFISH GILLNET DATA 1998–2008
Observed and Standardized CPUE (95% CI)



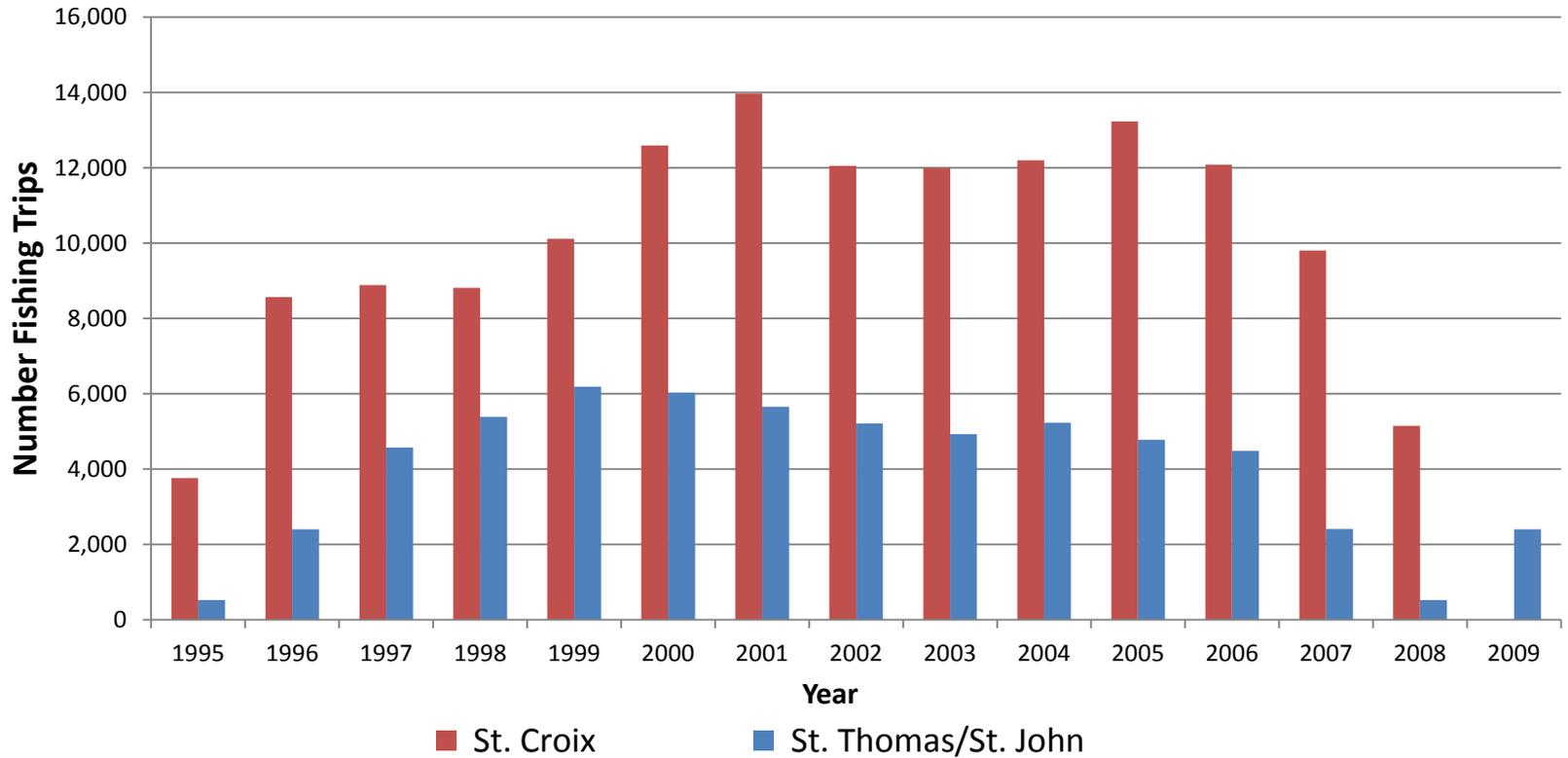
Parrotfish Family Nominal and Delta Lognormal CPUE in St. Croix

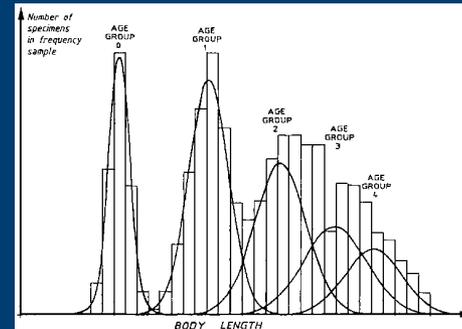
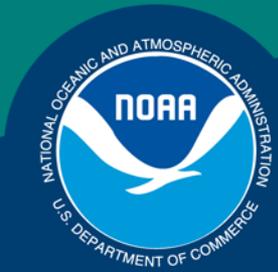


Effort Trends

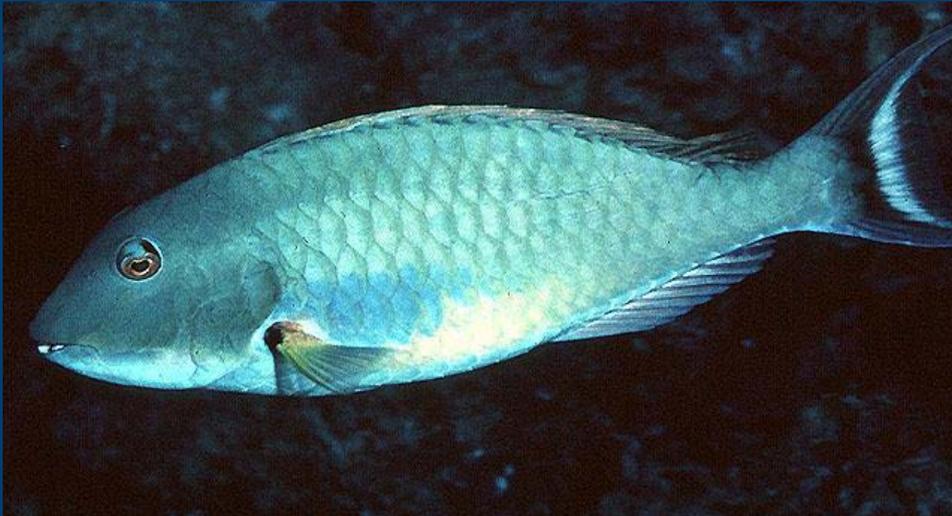
US Virgin Islands

Reported Number Commercial Fishing Trips
US Virgin islands





Redtail Parrotfish - Analysis of size-frequency



**NOAA
FISHERIES
SERVICE**

Parrotfish Overview

Between 1983 and 2011, the TIP data set has 98,829 records for 9 species of parrotfish and one aggregated parrotfish category. Due to either being landed on multiple-gear trips or not meeting length-weight criteria (outliers) another 24,954 parrotfish records were removed from the analysis.

Parrotfish

Parrotfish species total observations by island for the years 1983-2011.

	All	Puerto Rico	St Croix	St Thomas/St John
Redtail parrotfish	44184 (44%)	12105 (39%)	30411 (47%)	1599 (45%)
Stoplight parrotfish	36664 (37%)	14554 (46%)	20702 (32%)	1391(39%)
Redband parrotfish	7477 (7.5%)	845 (2.7%)	6406 (10%)	226 (6.4%)
Princess parrotfish	4054 (4.1%)	1275 (4.1%)	2680 (4.1%)	96 (2.7%)
Parrotfish spp	2728 (2.7%)	838 (2.7%)	1819 (2.8%)	71 (2%)
Redfin parrotfish	1804 (1.8 %)	174 (<1%)	1544 (2.4%)	86 (2%)
Queen parrotfish	1746 (1.7%)	1073 (3.4%)	659 (1%)	12 (<1%)
Rainbow parrotfish	77 (<1%)	61 (1.9%)	1 (<1%)	15 (<1%)
Blue parrotfish	72 (<1%)	58 (1.8%)	0	14 (<1%)
Striped parrotfish	23 (<1%)	0	19 (<1%)	4 (<1%)

* percent of column total shown in parentheses

Yellowtail Parrotfish (*Sparisoma rubripinne*)

Also known as: Redfin Parrotfish

Size: 8 to 14 ft. (20 to 35 cm) Depth: 3-35 ft. (1-12 m)

Distribution: Caribbean, Bahamas, Florida, Bermuda



Redtail Parrotfish - Terminal Phase

Sparisoma chrysopterus

Size: 14 to 16 in. (35 to 40 cm) Depth: 5-40 ft. (2-12 m)



Parrotfish

Parrotfish species total observations by island for the years 1983-2011.

	All	Puerto Rico	St Croix	St Thomas/St John
Redtail parrotfish	44184 (44%)	12105 (39%)	30411 (47%)	1599 (45%)
Stoptlight parrotfish	36664 (37%)	14554 (46%)	20702 (32%)	1391(39%)
Redband parrotfish	7477 (7.5%)	845 (2.7%)	6406 (10%)	226 (6.4%)
Princess parrotfish	4054 (4.1%)	1275 (4.1%)	2680 (4.1%)	96 (2.7%)
Parrotfish spp	2728 (2.7%)	838 (2.7%)	1819 (2.8%)	71 (2%)
Redfin parrotfish	1804 (1.8 %)	174 (<1%)	1544 (2.4%)	86 (2%)
Queen parrotfish	1746 (1.7%)	1073 (3.4%)	659 (1%)	12 (<1%)
Rainbow parrotfish	77 (<1%)	61 (1.9%)	1 (<1%)	15 (<1%)
Blue parrotfish	72 (<1%)	58 (1.8%)	0	14 (<1%)
Striped parrotfish	23 (<1%)	0	19 (<1%)	4 (<1%)

* percent of column total shown in parentheses

Redtail Parrotfish

PUERTO RICO	GILL NETS	7242
PUERTO RICO	POTS AND TRAP	4180
PUERTO RICO	BY HAND	235
PUERTO RICO	HAUL SEINES	210
PUERTO RICO	HAND LINE	190
PUERTO RICO		32
PUERTO RICO	HOOK AND LINE	9
PUERTO RICO	OTHER GEARS	5
PUERTO RICO	SPEARS AND GI	1
ST CROIX	POTS AND TRAP	23439
ST CROIX	GILL NETS	5530
ST CROIX	SPEARS AND GI	730
ST CROIX	BY HAND	462
ST CROIX	LONG LINES	222
ST CROIX	HAND LINE	17
ST THOMAS/ST JOHN	POTS AND TRAP	1565
ST THOMAS/ST JOHN	HAUL SEINES	18
ST THOMAS/ST JOHN		15
ST THOMAS/ST JOHN	HAND LINE	1

Resulting sample sizes for island and gear combinations following DW guidance to combine specific gear categories.

The analysis type is also indicated (TS indicates a length-based time-series analysis was done and ID indicates insufficient data for analysis). Note: Sample sizes are reduced from previous slide due to QA/QC and removal of outliers and samples from multi-gear trips.

SPECIES NAME	ISLAND	GEAR NAME	SAMPLES	ANALYSIS
REDTAIL PARROTFISH	PUERTO RICO	DIVERS	199	ID
REDTAIL PARROTFISH	PUERTO RICO	HOOK AND LINE	183	ID
REDTAIL PARROTFISH	PUERTO RICO	NETS	6853	TS
REDTAIL PARROTFISH	PUERTO RICO	POTS AND TRAPS	3693	TS
REDTAIL PARROTFISH	ST CROIX	DIVERS	3111	TS
REDTAIL PARROTFISH	ST CROIX	HOOK AND LINE	239	ID
REDTAIL PARROTFISH	ST CROIX	NETS	6761	TS
REDTAIL PARROTFISH	ST CROIX	POTS AND TRAPS	25393	TS
REDTAIL PARROTFISH	ST THOMAS	HOOK AND LINE	1	ID
REDTAIL PARROTFISH	ST THOMAS	NETS	42	ID
REDTAIL PARROTFISH	ST THOMAS	POTS AND TRAPS	1481	TS

Resulting sample sizes for island and gear combinations following DW guidance to combine specific gear categories.

The analysis type is also indicated (TS indicates a length-based time-series analysis was done and ID indicates insufficient data for analysis). Note: Sample sizes are reduced from previous slide due to QA/QC and removal of outliers and samples from multi-gear trips.

SPECIES NAME	ISLAND	GEAR NAME	SAMPLES	ANALYSIS
REDTAIL PARROTFISH	PUERTO RICO	DIVERS	199	ID
REDTAIL PARROTFISH	PUERTO RICO	HOOK AND LINE	183	ID
REDTAIL PARROTFISH	PUERTO RICO	NETS	6853	TS
REDTAIL PARROTFISH	PUERTO RICO	POTS AND TRAPS	3693	TS
REDTAIL PARROTFISH	ST CROIX	DIVERS	3111	TS
REDTAIL PARROTFISH	ST CROIX	HOOK AND LINE	239	ID
REDTAIL PARROTFISH	ST CROIX	NETS	6761	TS
REDTAIL PARROTFISH	ST CROIX	POTS AND TRAPS	25393	TS
REDTAIL PARROTFISH	ST THOMAS	HOOK AND LINE	1	ID
REDTAIL PARROTFISH	ST THOMAS	NETS	42	ID
REDTAIL PARROTFISH	ST THOMAS	POTS AND TRAPS	1481	TS

Redtail parrotfish, *Sparisoma chrysopteron*

Location	Reference	n	Sex	Age-length relationship						Length-weight relationship			Maturity	
				t_0	L_{inf}	K	M	L_{max}	t_{max}	a	b	W_{inf}	L_{mat}	t_{mat}
Central Western Atlantic	Choat & Robertson (2002)	124	-	-0.04	263	1.176	-	347	5	-	-	-	-	-
Guadeloupe, Martinique	Bouchon- Navarro et al. (2006)	38	-	-	-	-	-	406	-	0.015	3.042	-	242 ^o	-
Panama	Robertson & Warner (1978)	-	-	-	-	-	-	-	-	-	-	-	140 ^{†o}	-
Puerto Rico	Figuerola et al. (1998)	-	-	-	-	-	-	-	-	-	-	-	235 ^{†o}	-
South Florida	Molina-Urena (2009)	105	-	-	-	-	-	-	-	0.015	3.027	-	-	-

Model Inputs: Sensitivity range

Redtail parrotfish, Puerto Rico, Pot and traps

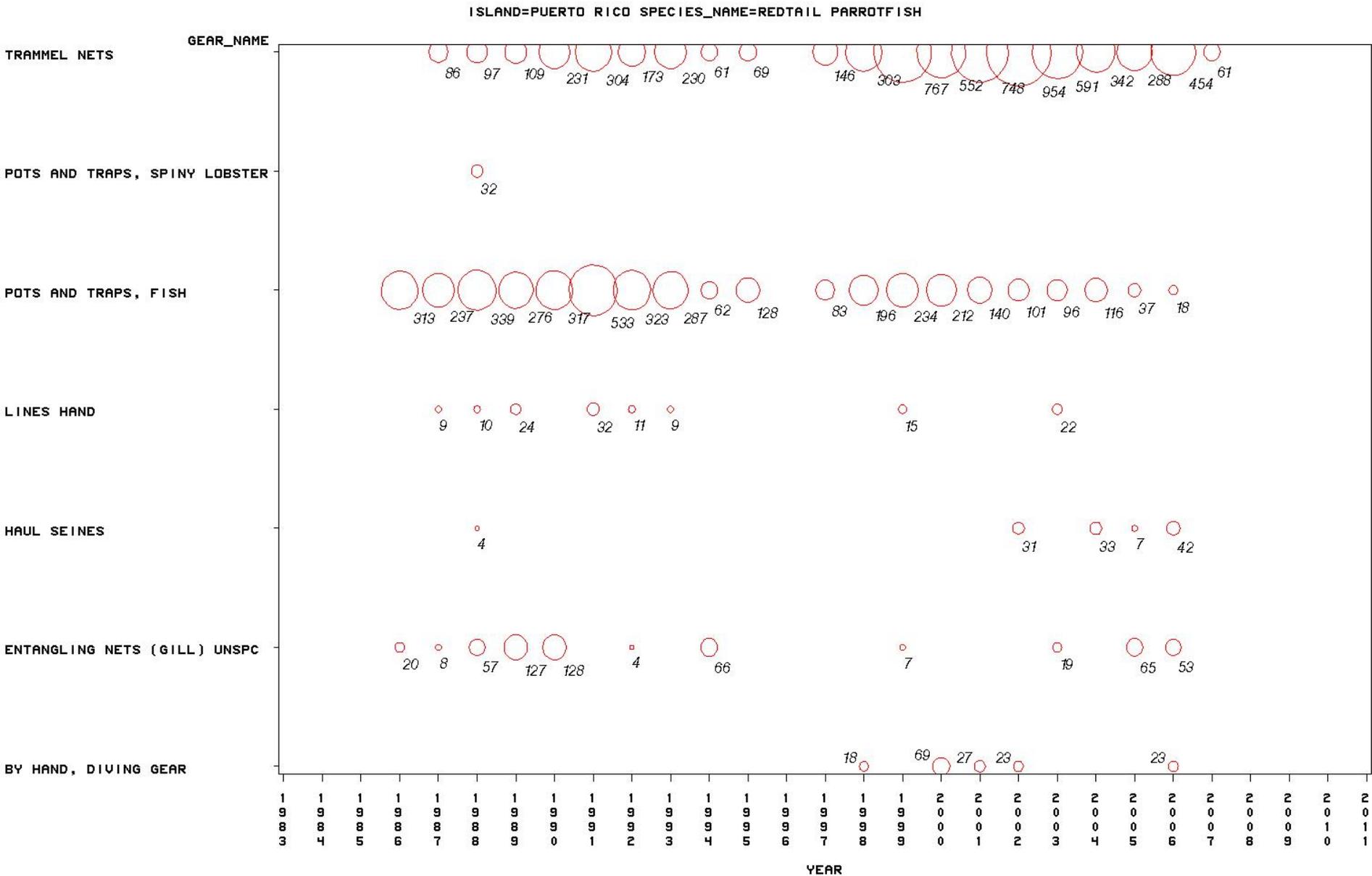
Parameter	Lower Bound	Base Case	Upper Bound
L_c	246mm	256mm	286mm
K	0.312	0.78	1.212
L_∞	270mm	300mm	390mm

Puerto Rico

Overview of Data

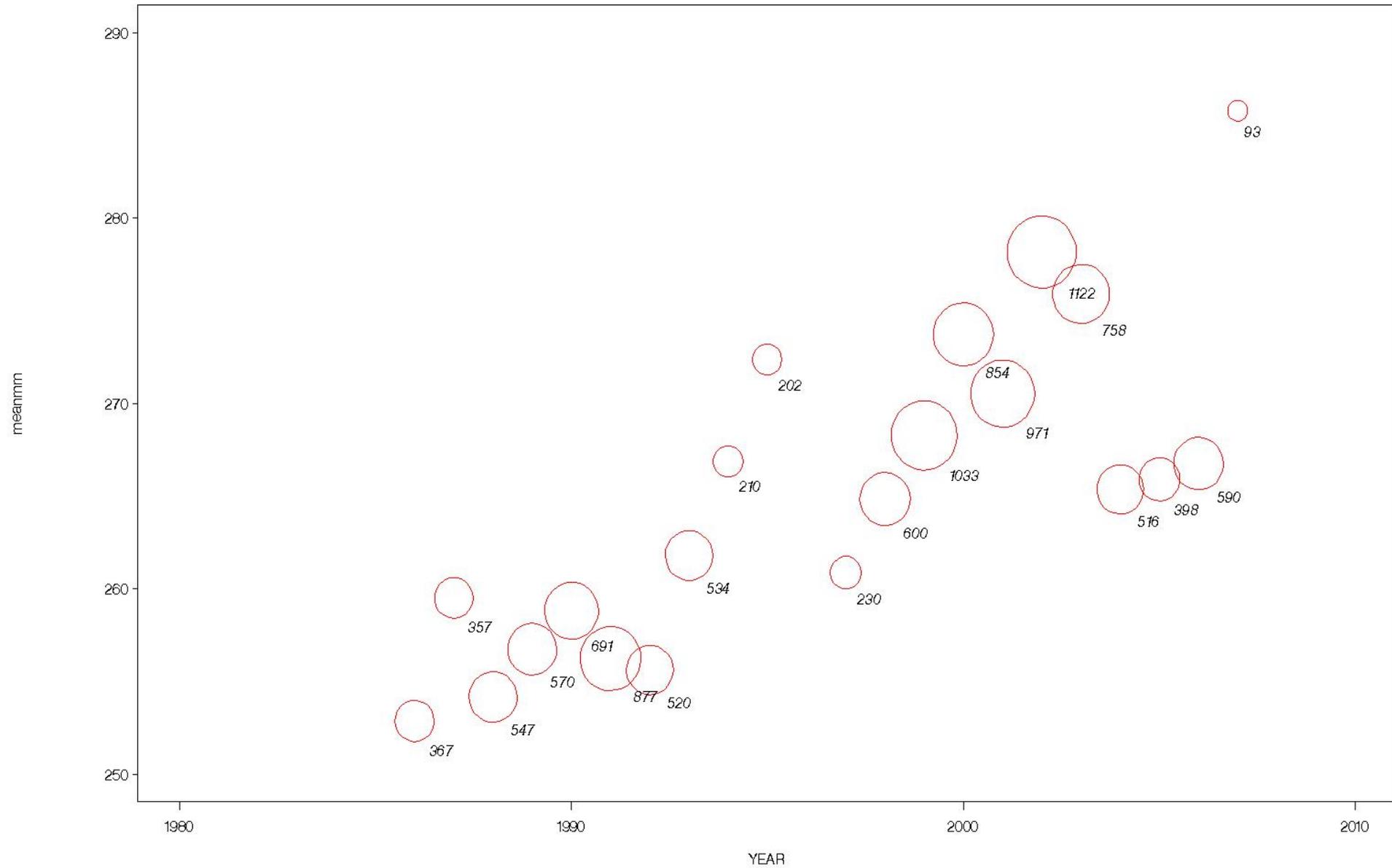
Redtail Parrotfish - Puerto Rico

Redtail Parrotfish number of sampled fish by specific gear type from Puerto Rico, 1983-2011.



Redtail parrotfish mean lengths and sample sizes from Puerto Rico, 1983-2011.

ISLAND= PUERTO RICO SPECIES_NAME= REDTAIL PARROTFISH

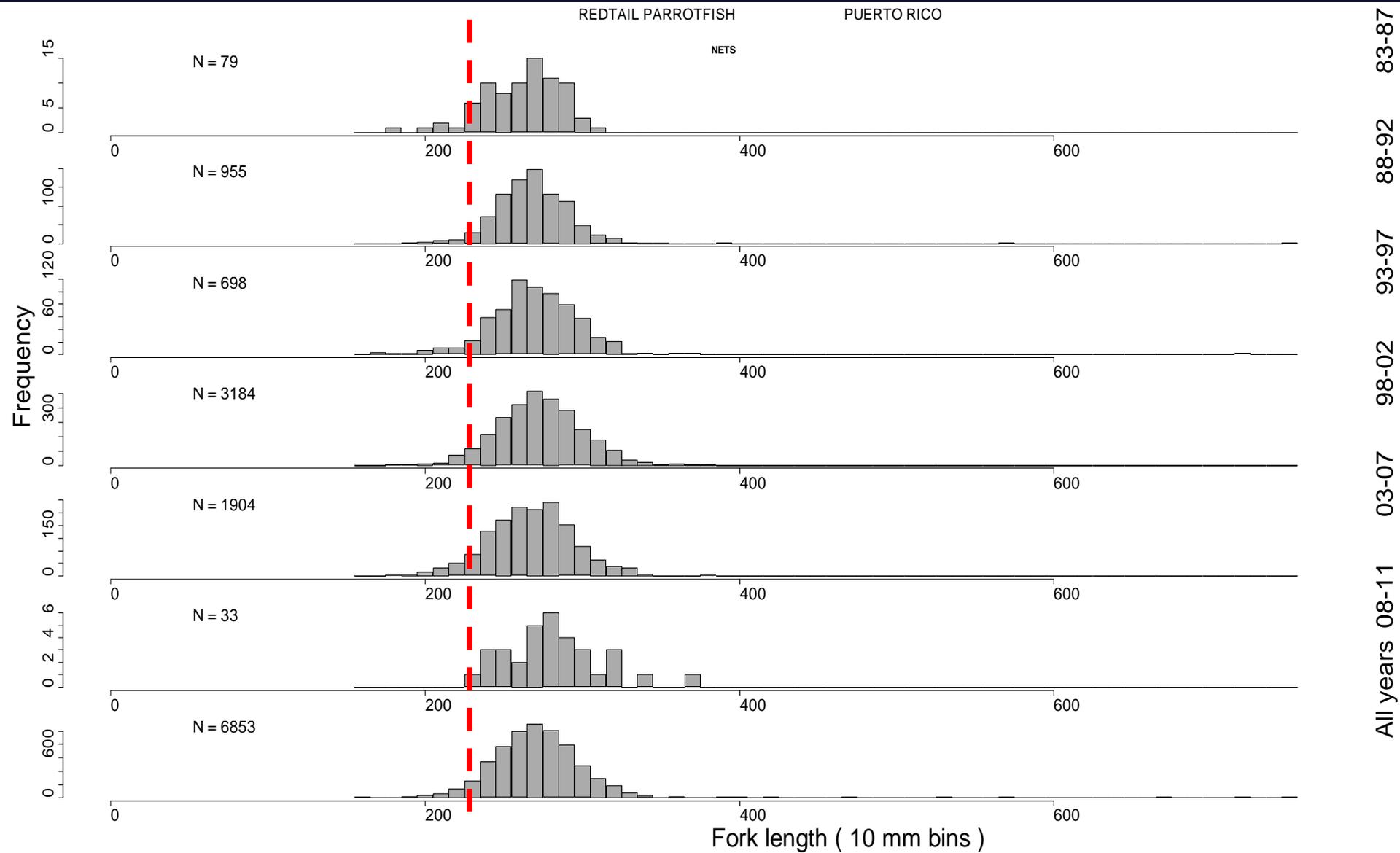


Redtail Parrotfish

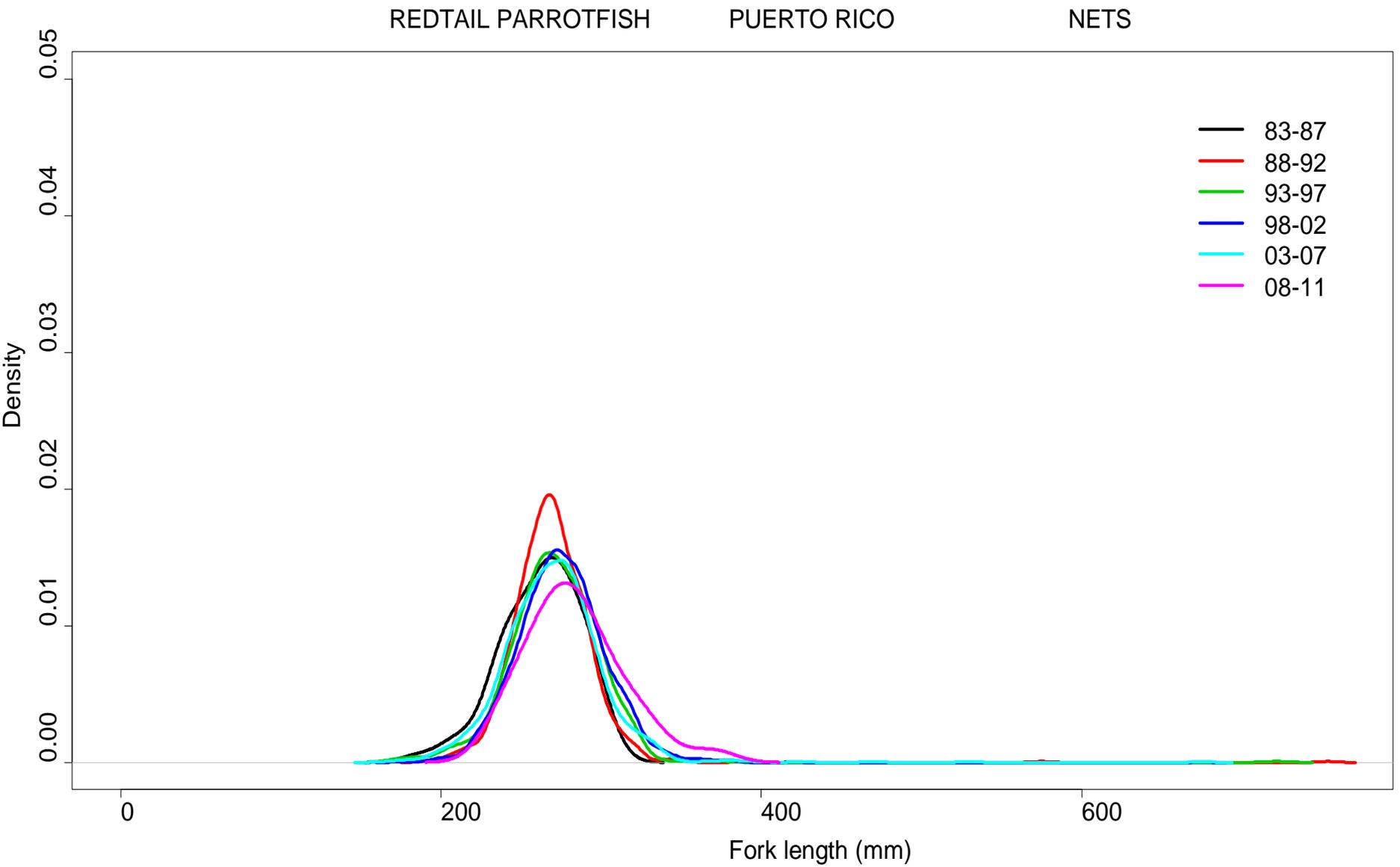
Puerto Rico

Nets

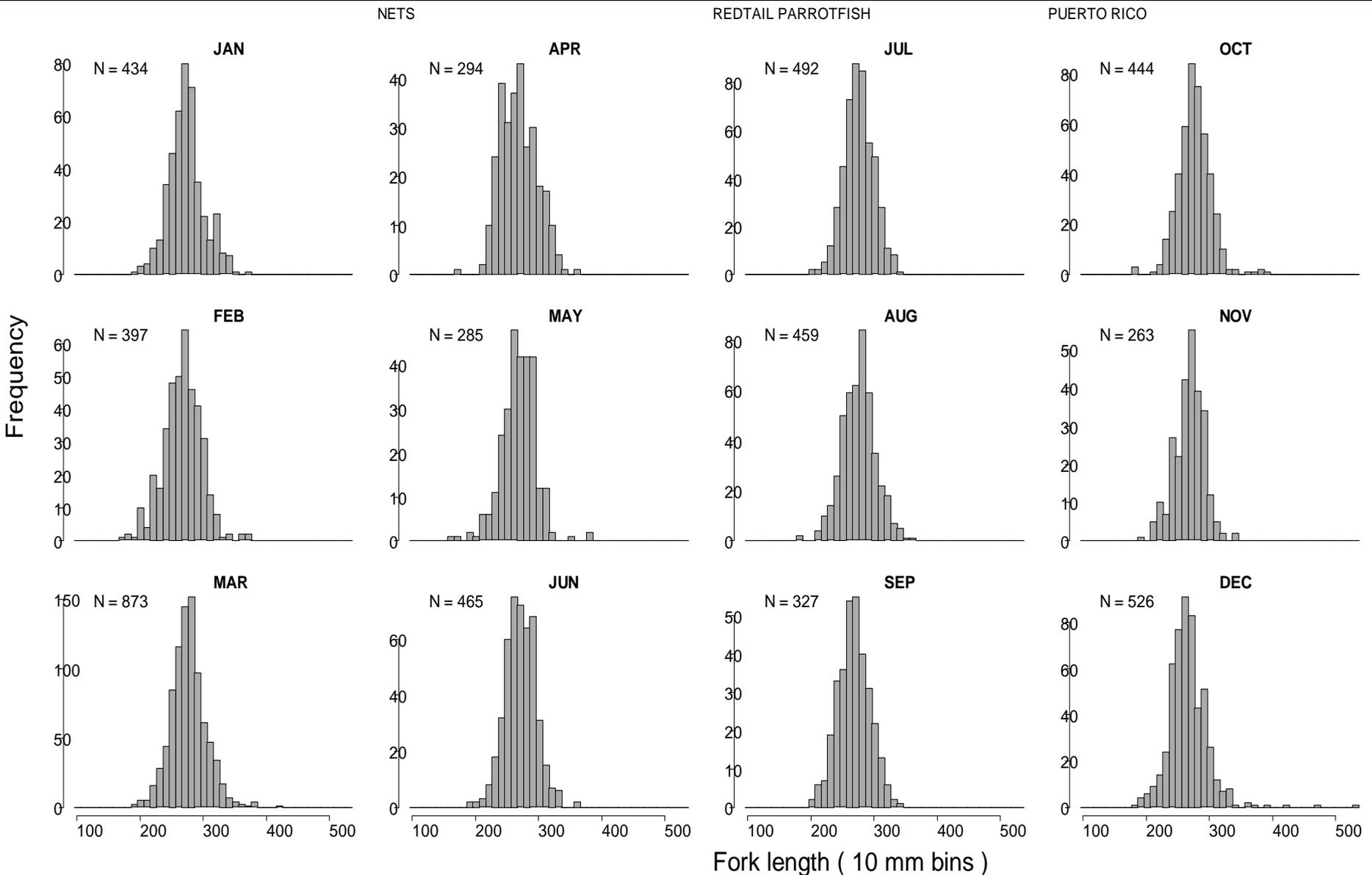
Length frequency distribution for redbtail parrotfish caught by nets in Puerto Rico.



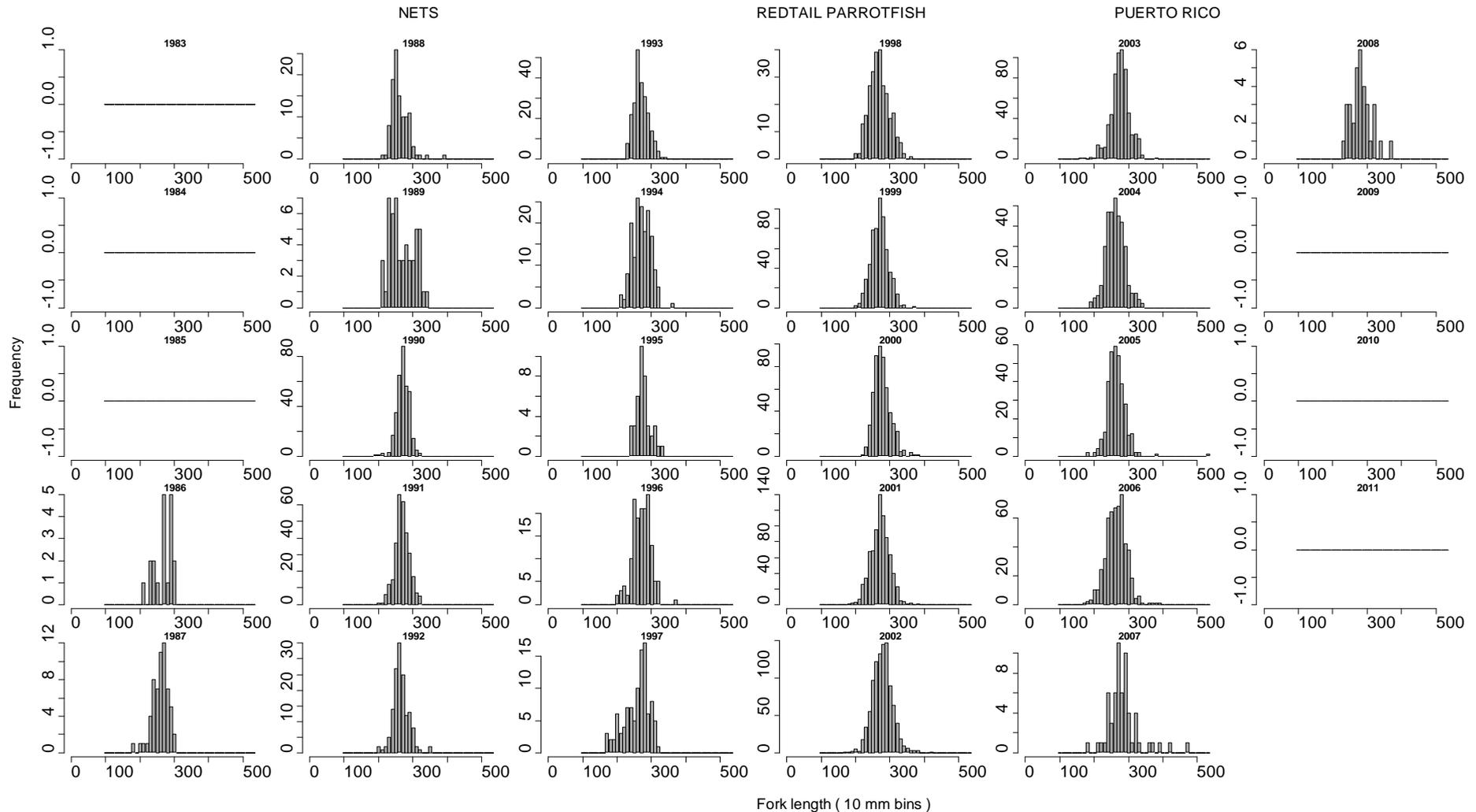
Density plot of observed lengths from the TIP database for redbtail parrotfish caught by nets in Puerto Rico. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redbtail parrotfish caught in the net fishery in Puerto Rico. N represents the sample size.

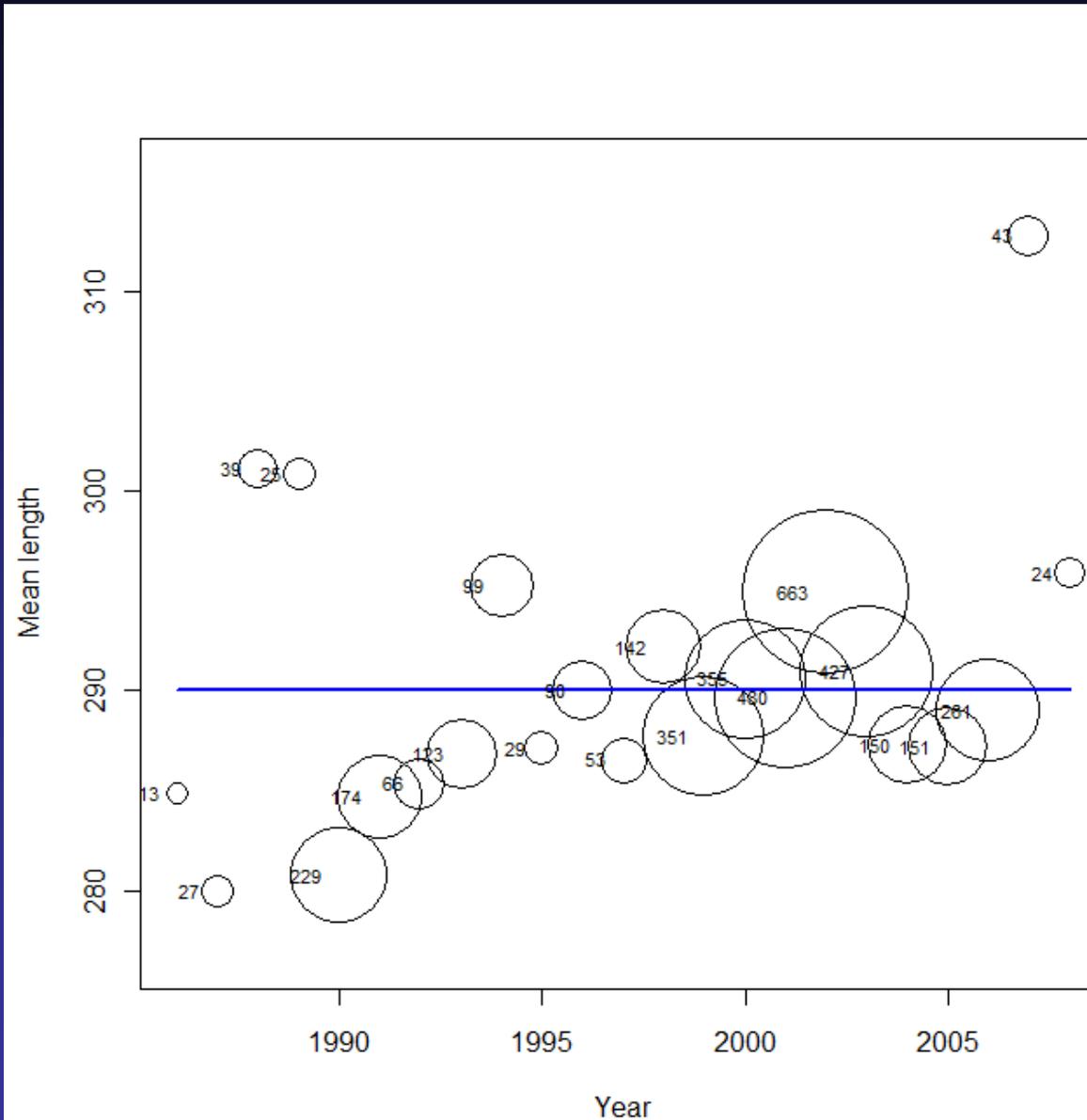


Annual length-frequency histograms for redbtail parrotfish caught in the net fishery in Puerto Rico. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



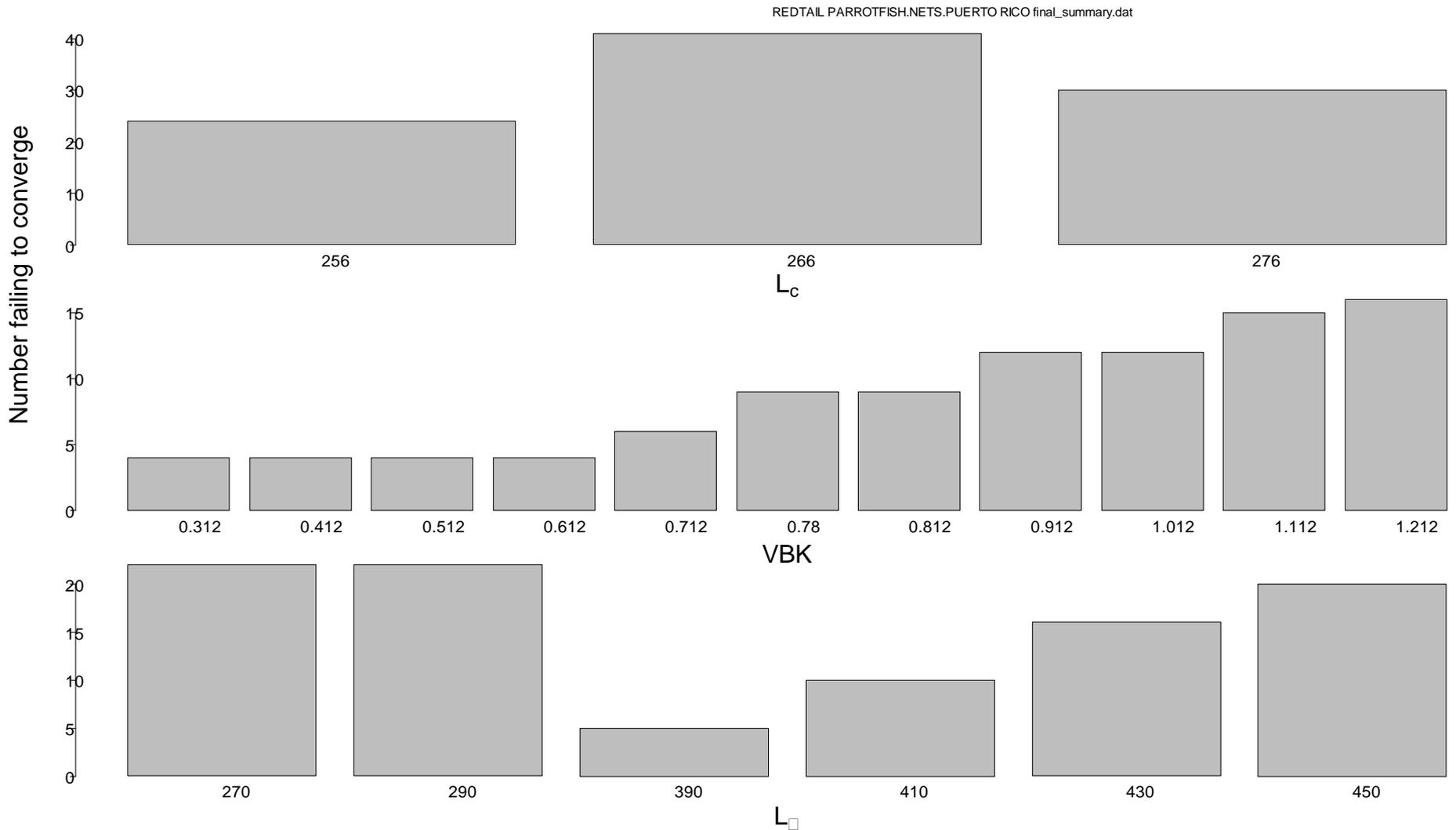
Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by nets in Puerto Rico.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.



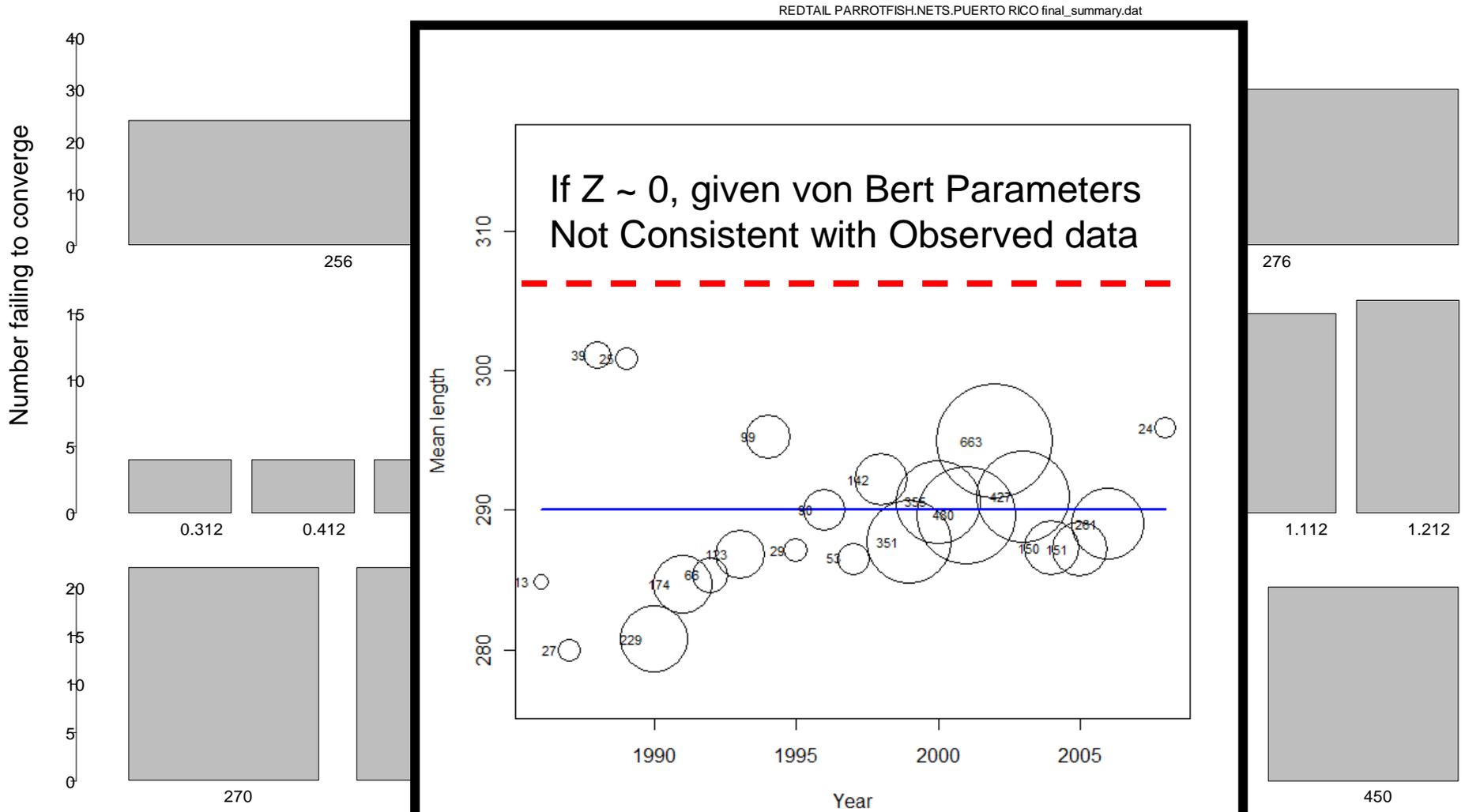
Sensitivity runs

Number of runs which failed to converge with respect to the input parameter values for the net fishery in Puerto Rico.



Sensitivity runs

Number of runs which failed to converge (e.g. hit bounds) with respect to the input parameter values for the net fishery in Puerto Rico.

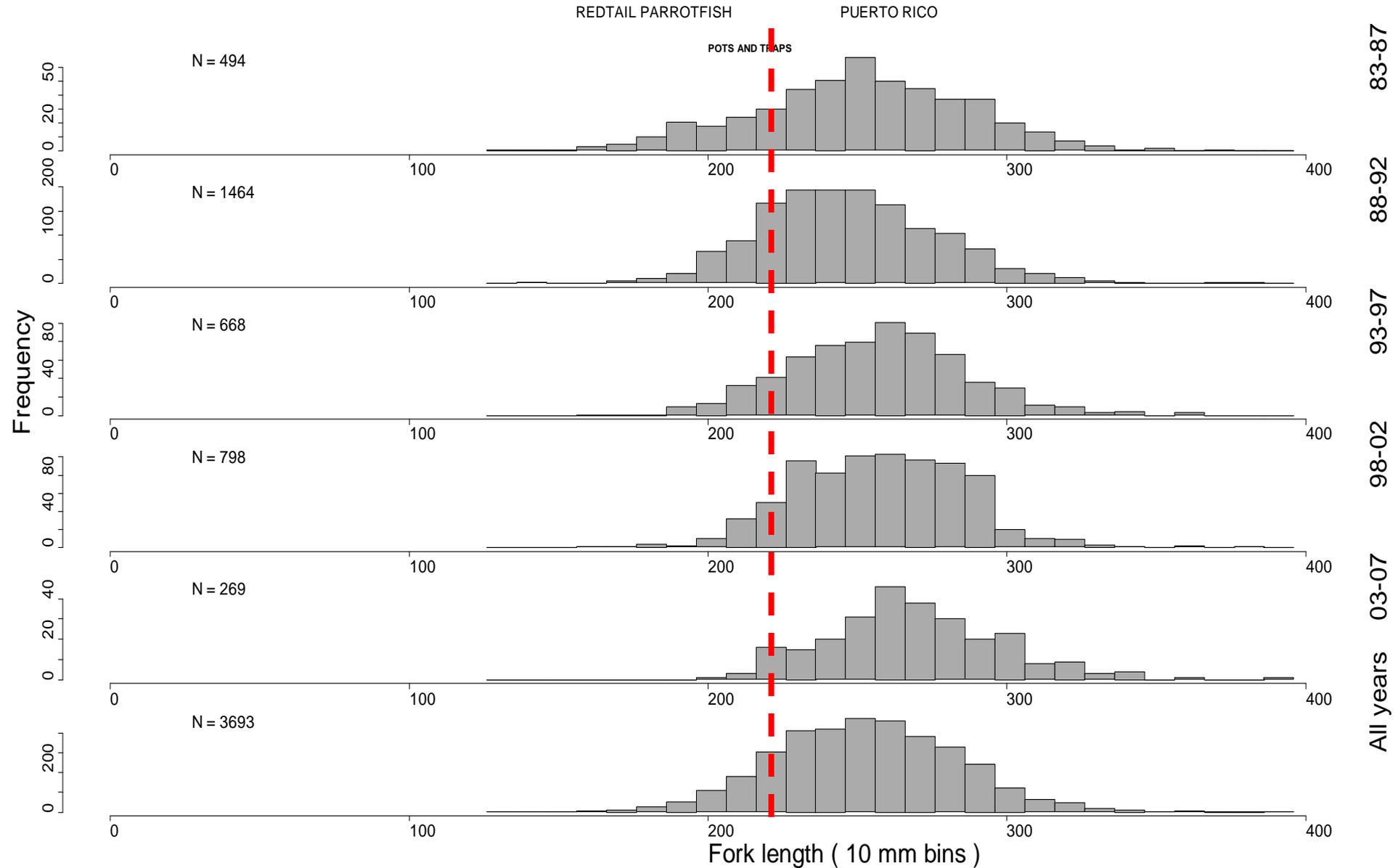


Redtail Parrotfish

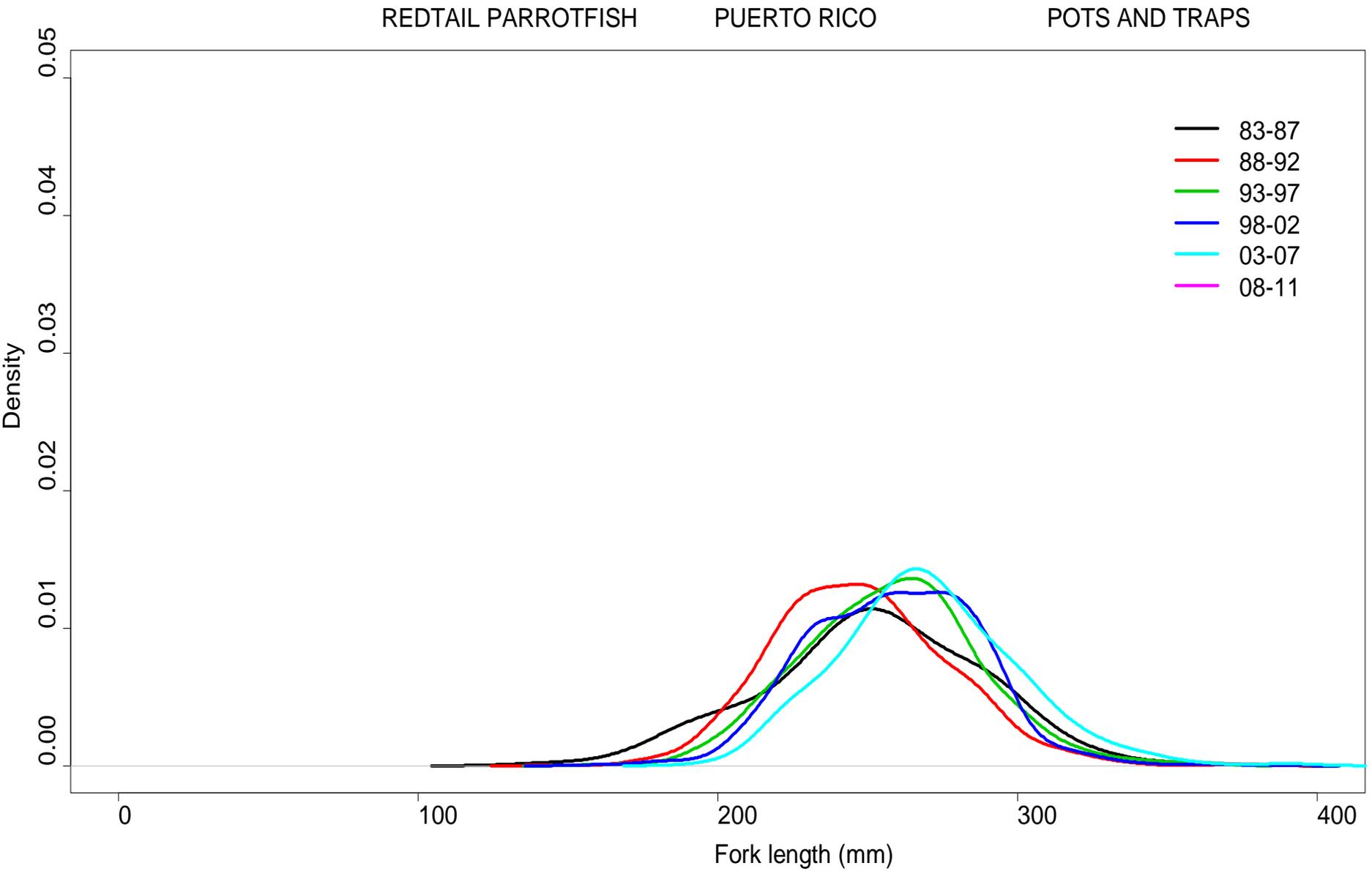
Puerto Rico

Pots and Traps

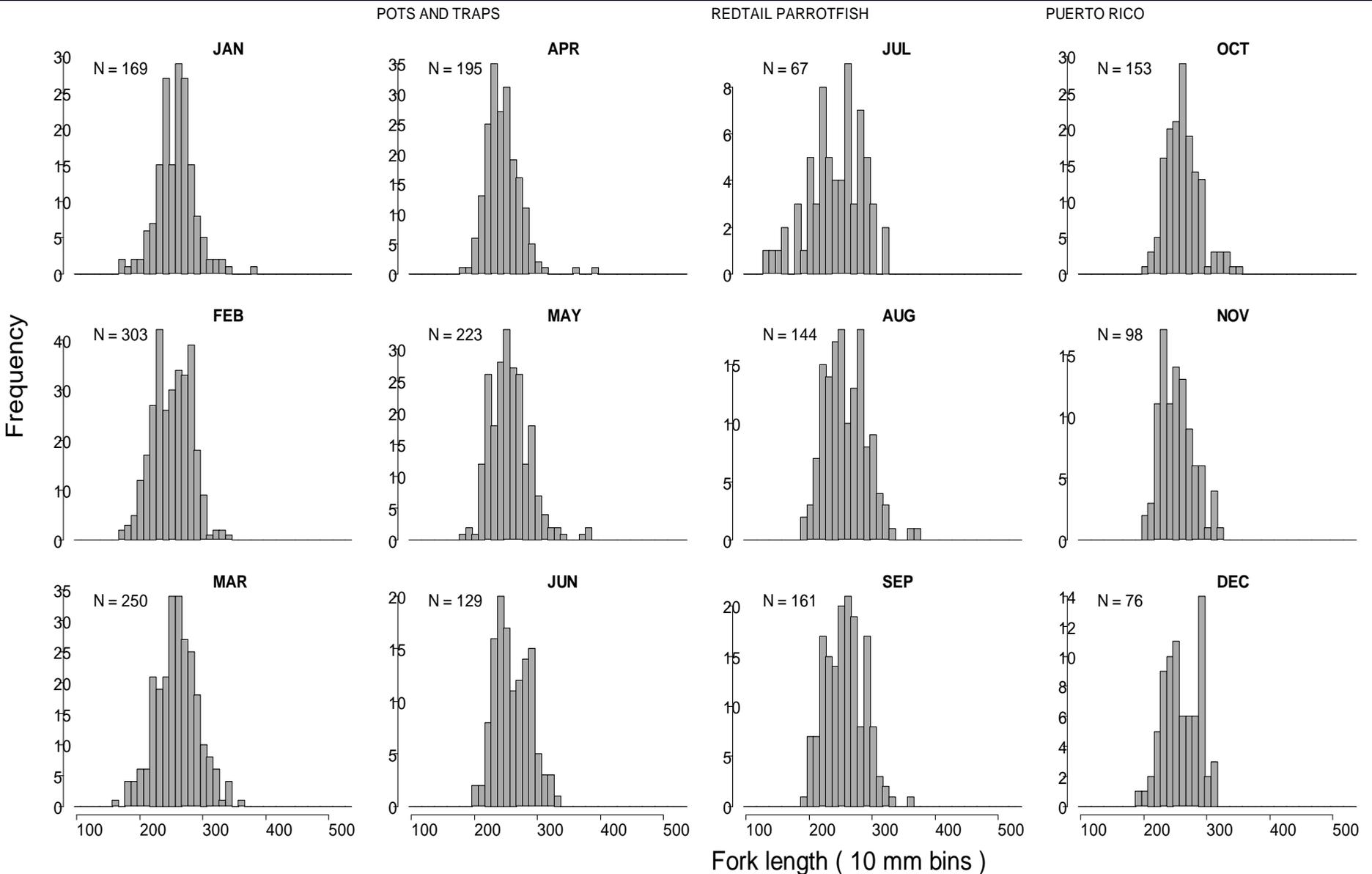
Length frequency distribution for redbtail parrotfish caught by pots and traps in Puerto Rico.



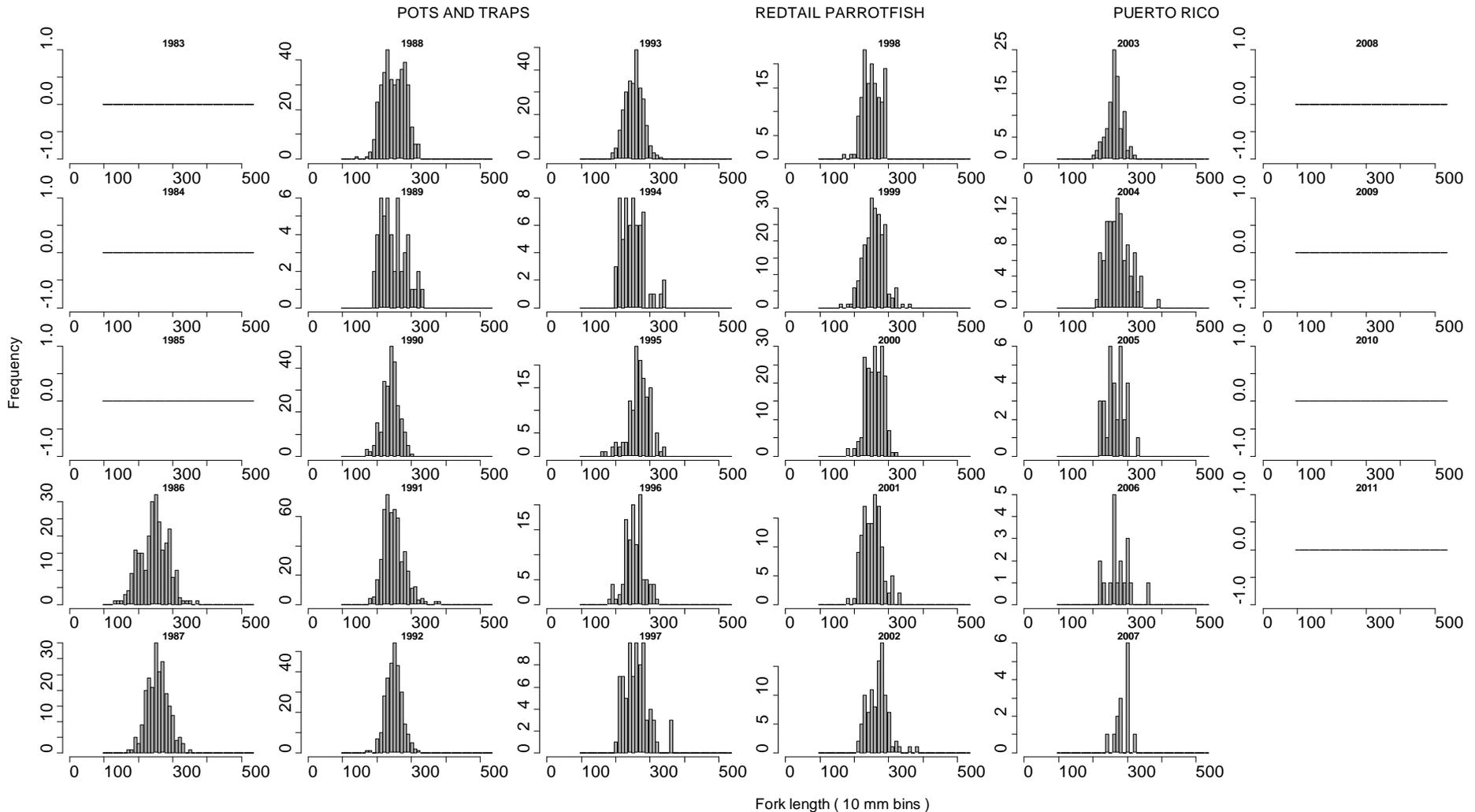
Density plot of observed lengths from the TIP database for redbtail parrotfish caught by pots and traps in Puerto Rico. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redbtail parrotfish caught the pot and trap fishery in Puerto Rico. N represents the sample size.

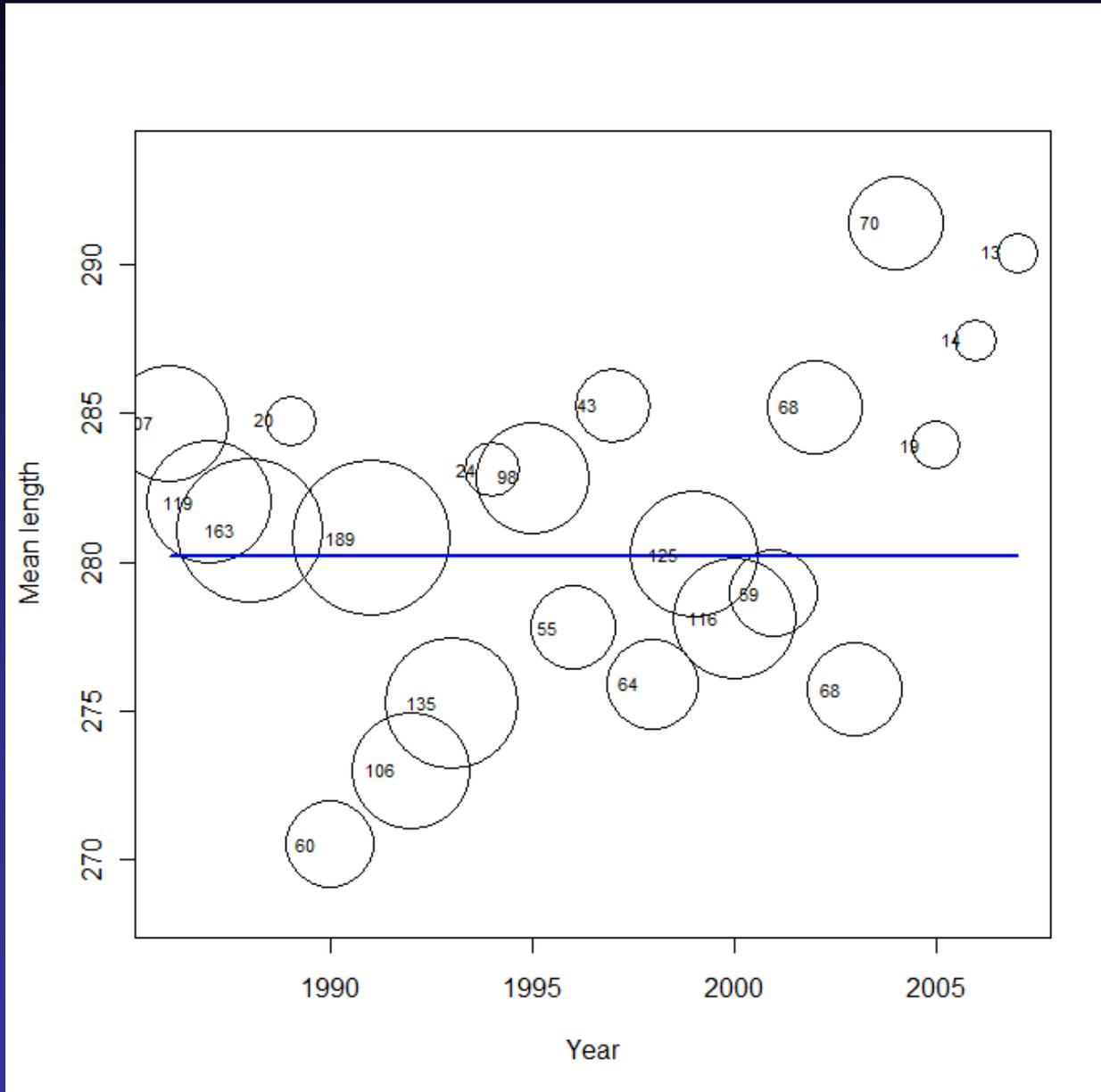


Annual length-frequency histograms for redbtail parrotfish caught by the pot and trap fishery in Puerto Rico. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by pots and traps in Puerto Rico.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

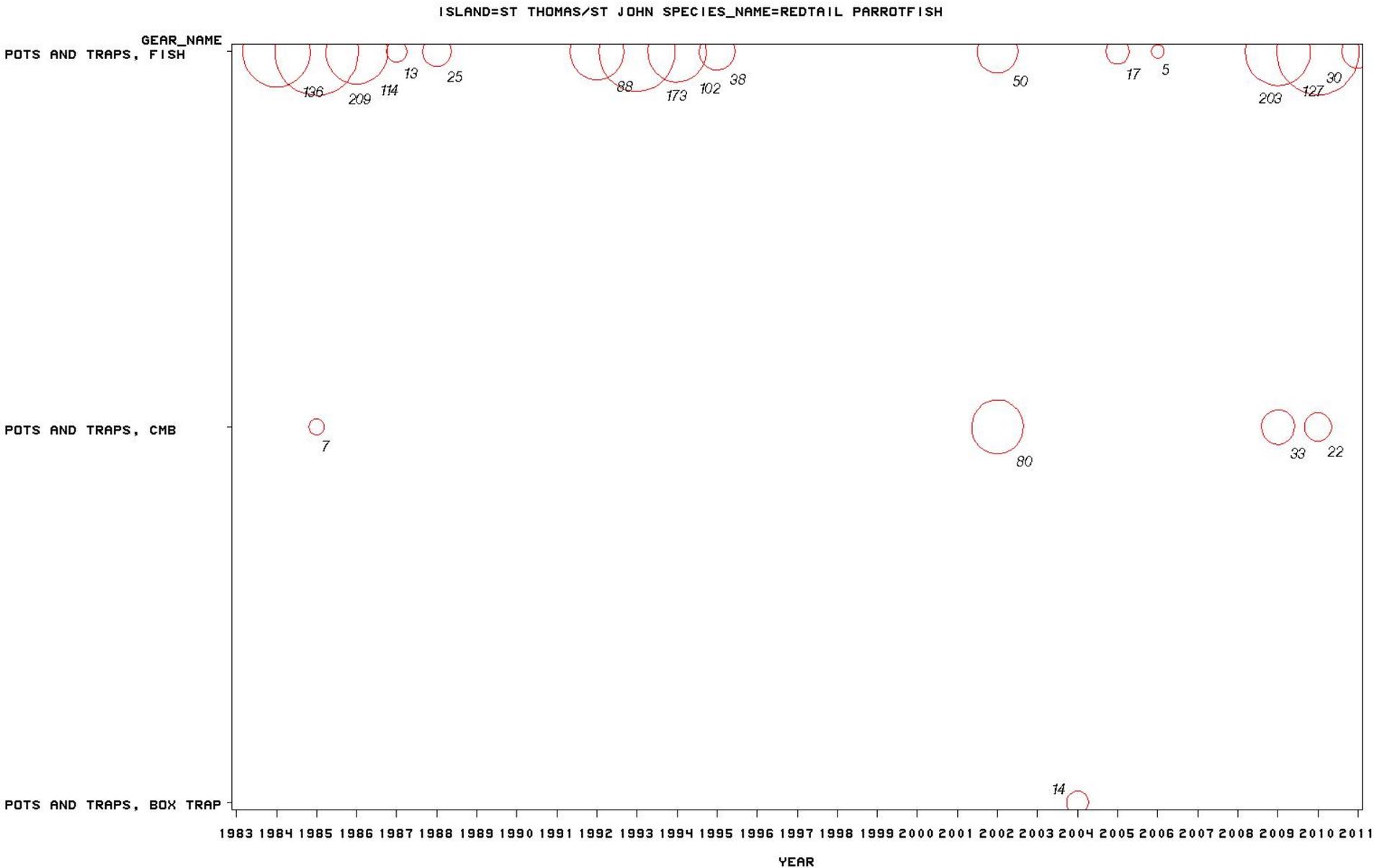


St. Thomas/St. John

Overview of Data

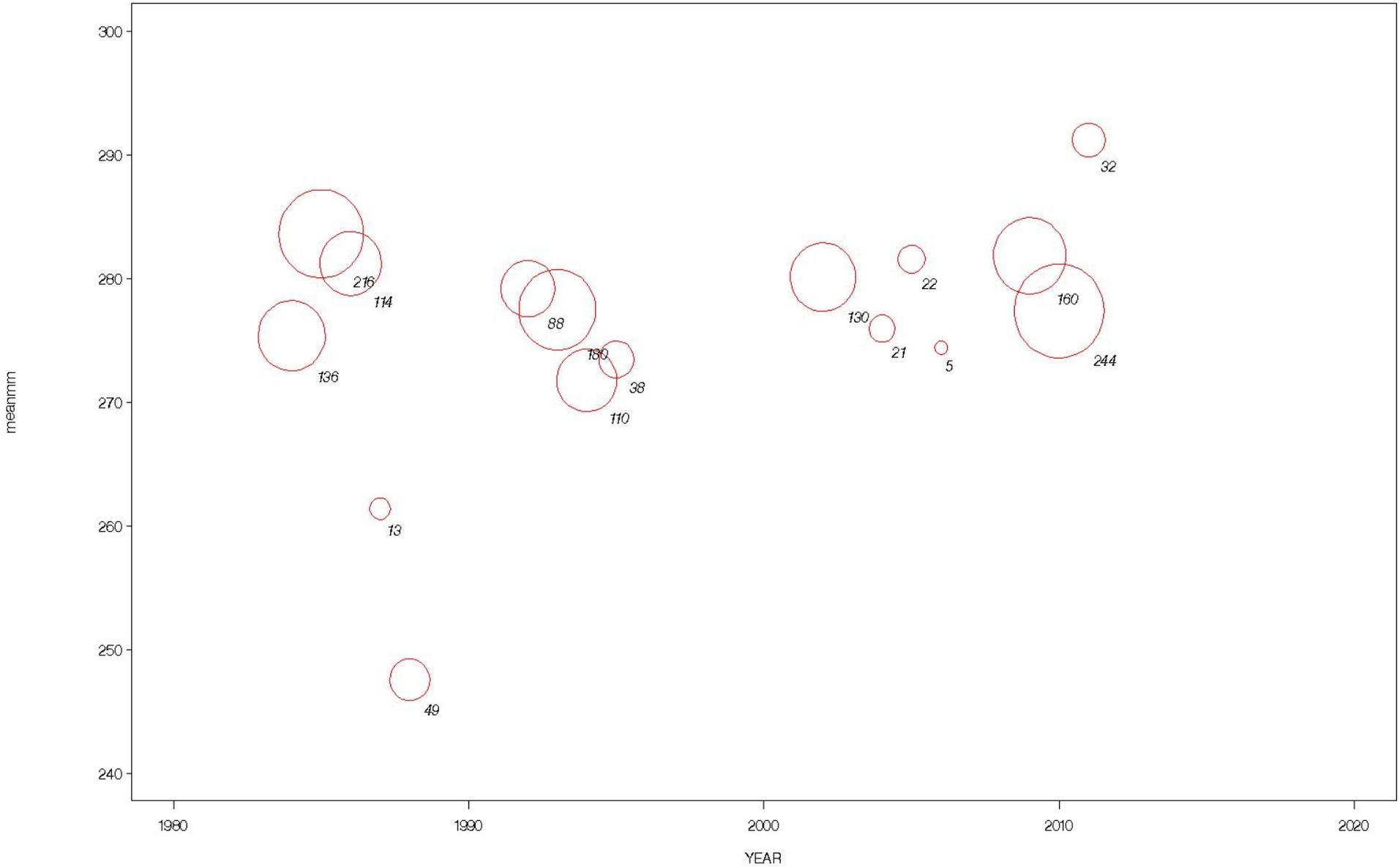
Redtail Parrotfish – St. Thomas/St. John

Redtail Parrotfish number of sampled fish by specific gear type from St. Thomas/St. John, 1983-2011



Redtail parrotfish mean lengths and sample sizes from St. Thomas – St. John, 1983-2011.

ISLAND= ST THOMAS/ST JOHN SPECIES_NAME= REDTAIL PARROTFISH

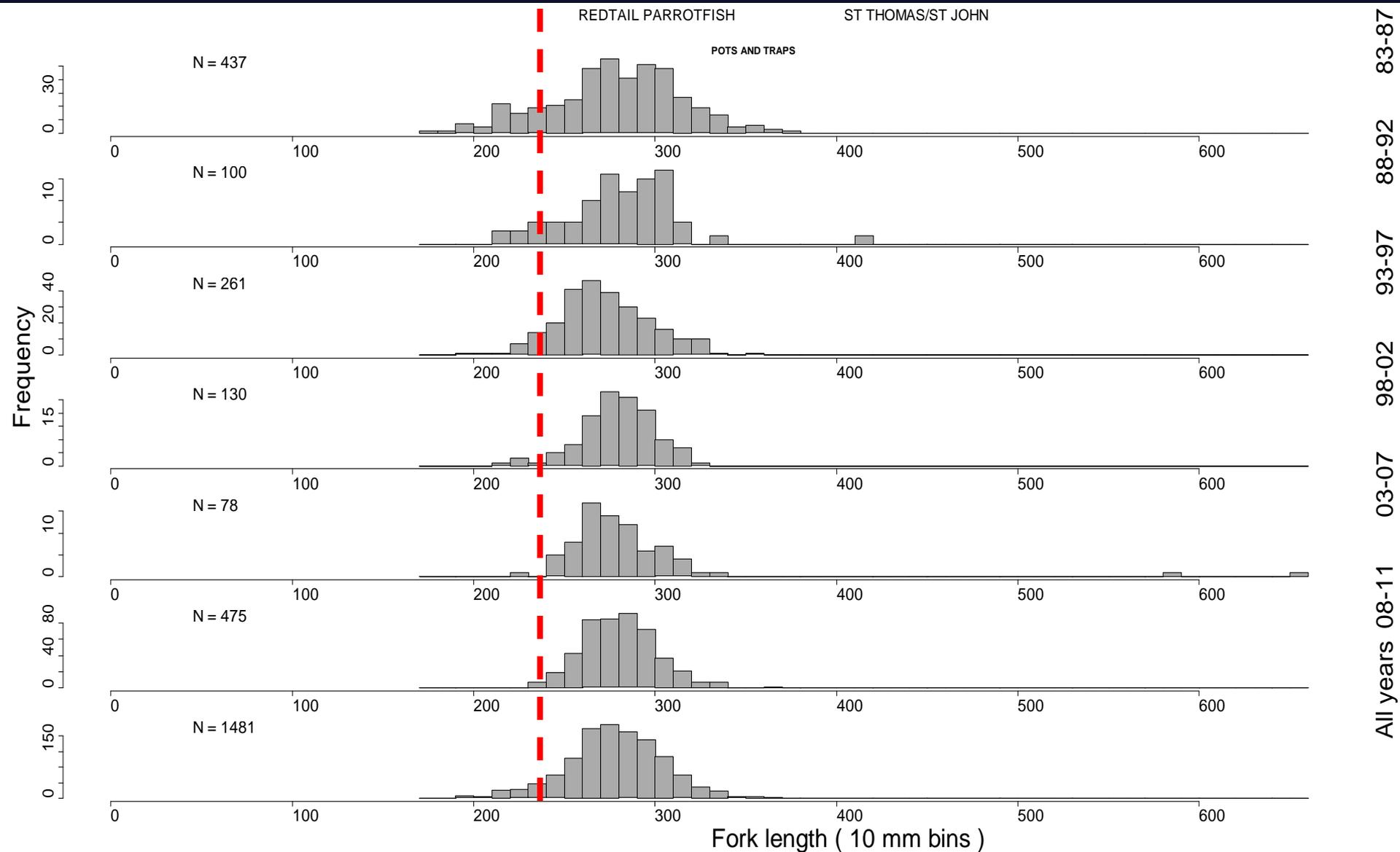


Redtail Parrotfish

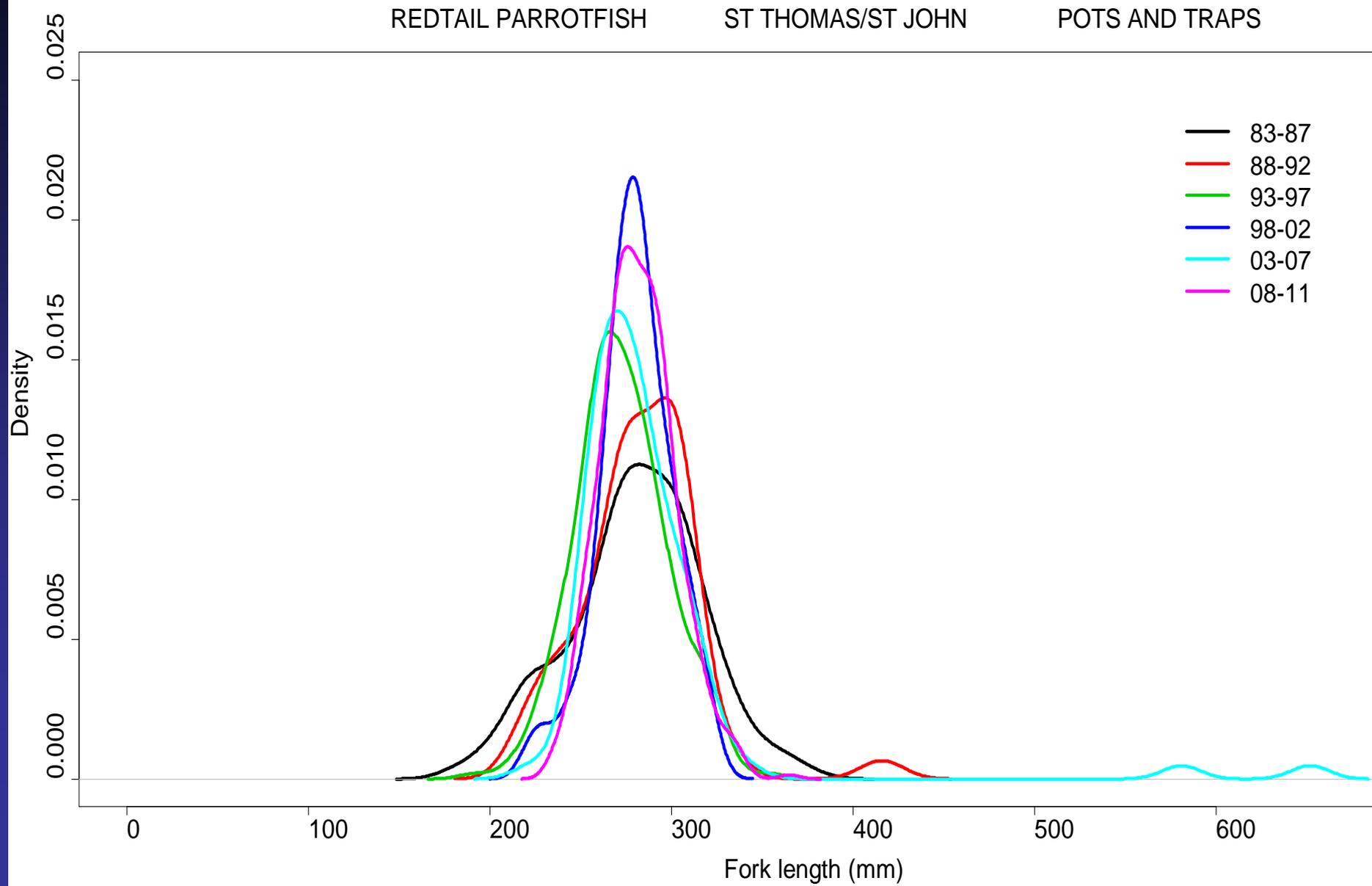
St. Thomas/St. John

Pots and Traps

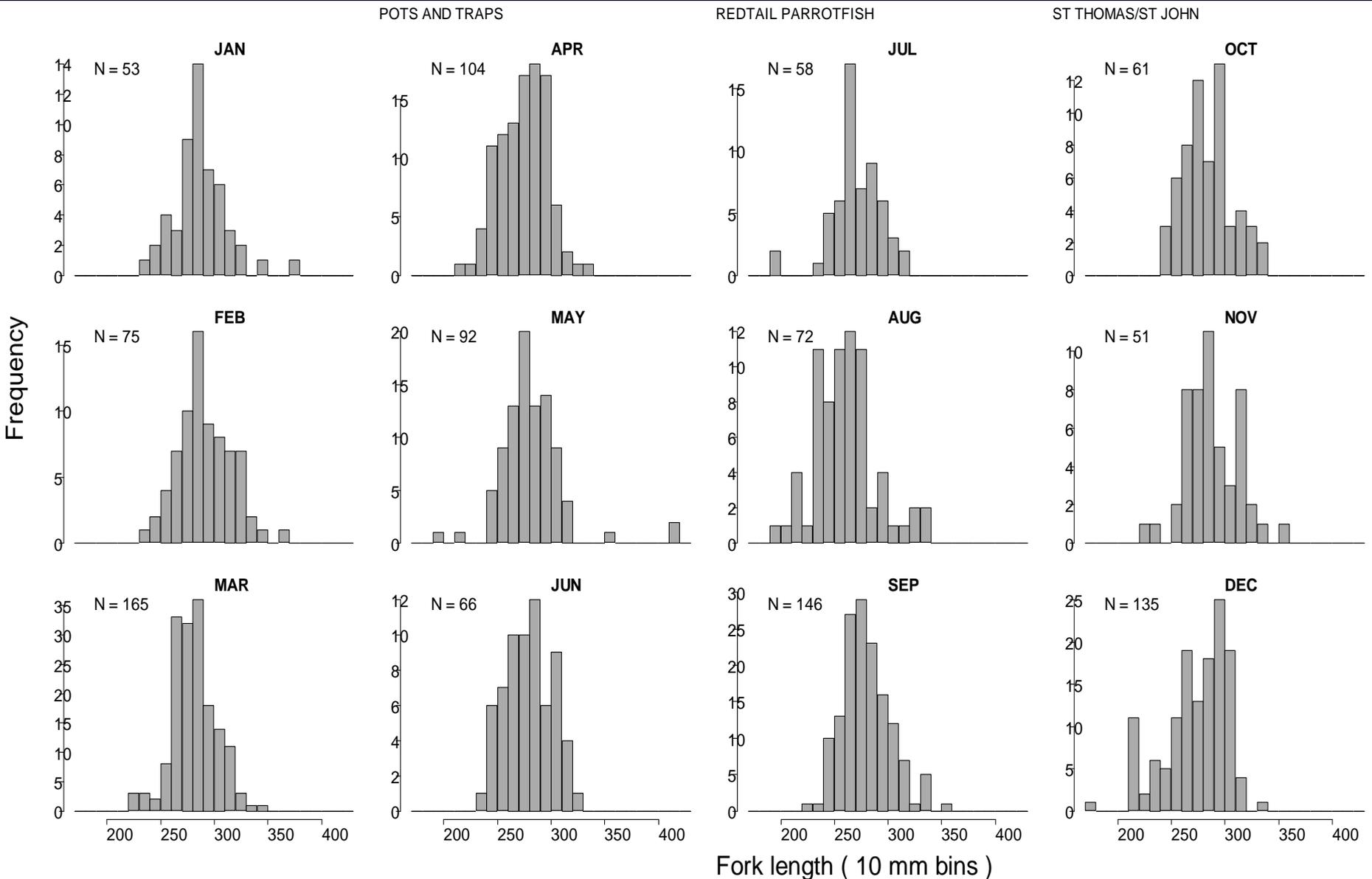
Length frequency distribution for redbtail parrotfish caught by pots and traps in St. Thomas/St. John.



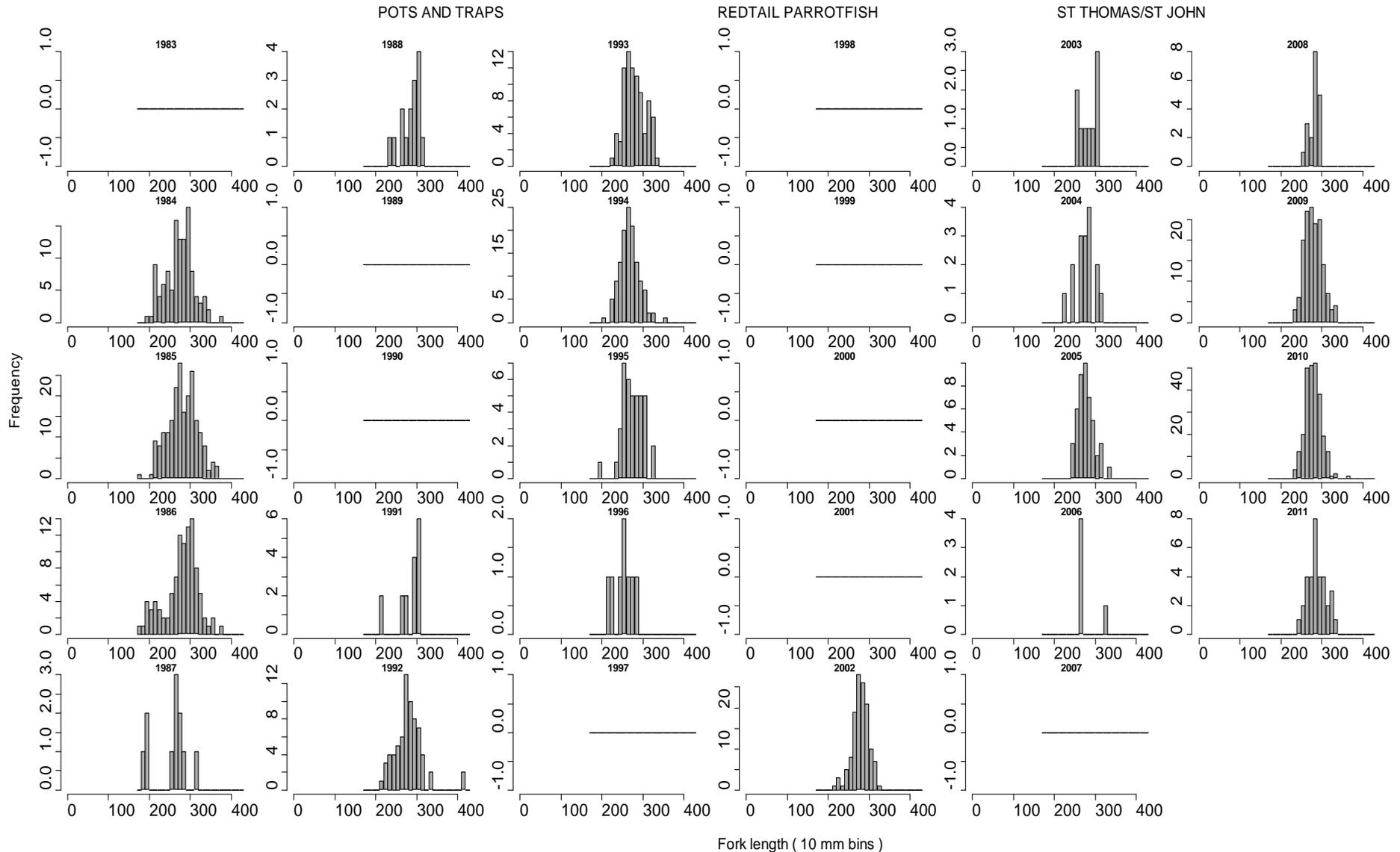
Density plot of observed lengths from the TIP database for redtail parrotfish caught by pots and traps in St. Thomas/St. John. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redbtail parrotfish caught the pot and trap fishery in St. Thomas/St. John. N represents the sample size.

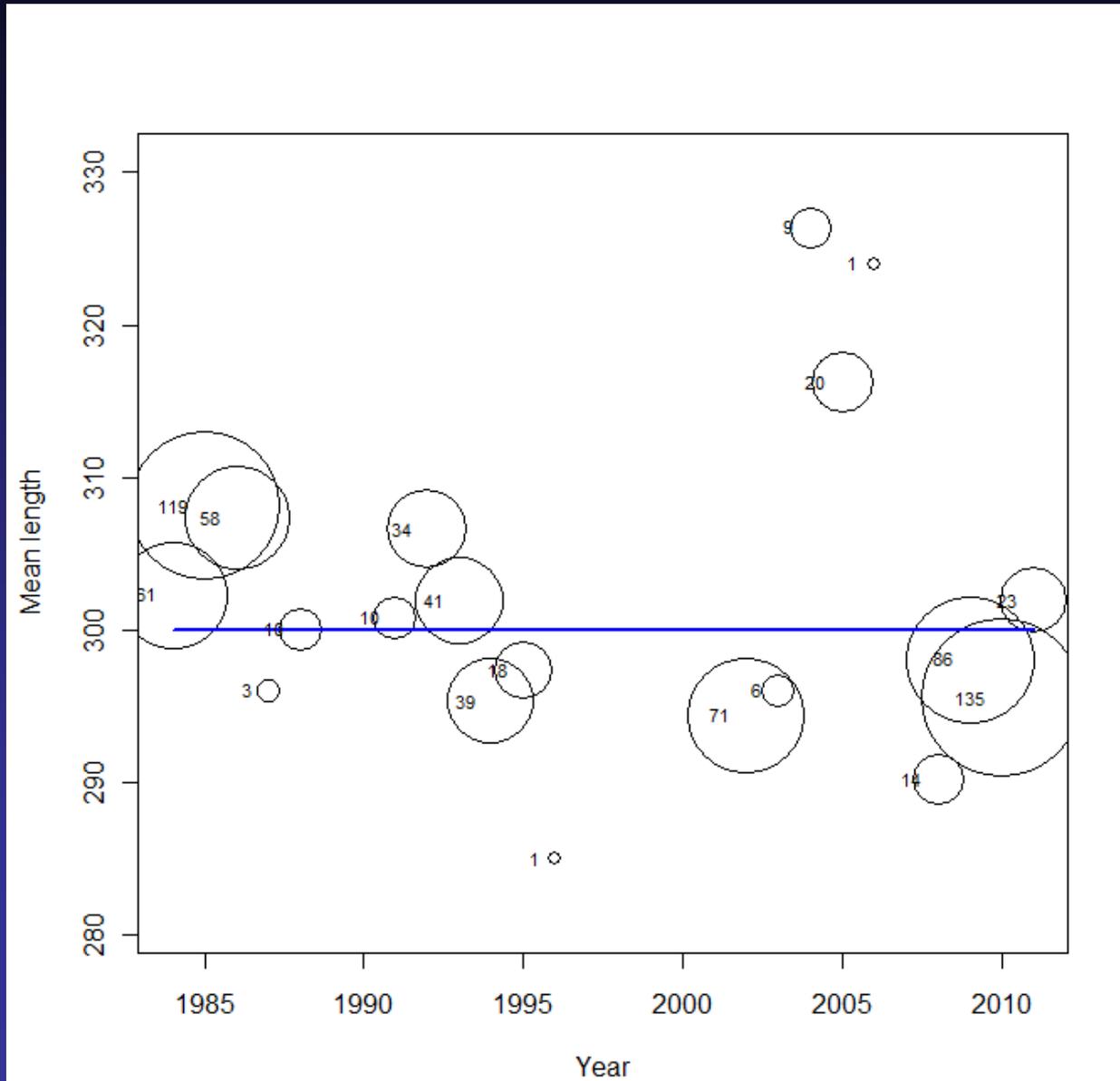


Annual length-frequency histograms for redbtail parrotfish caught by the pot and trap fishery in Puerto Rico. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



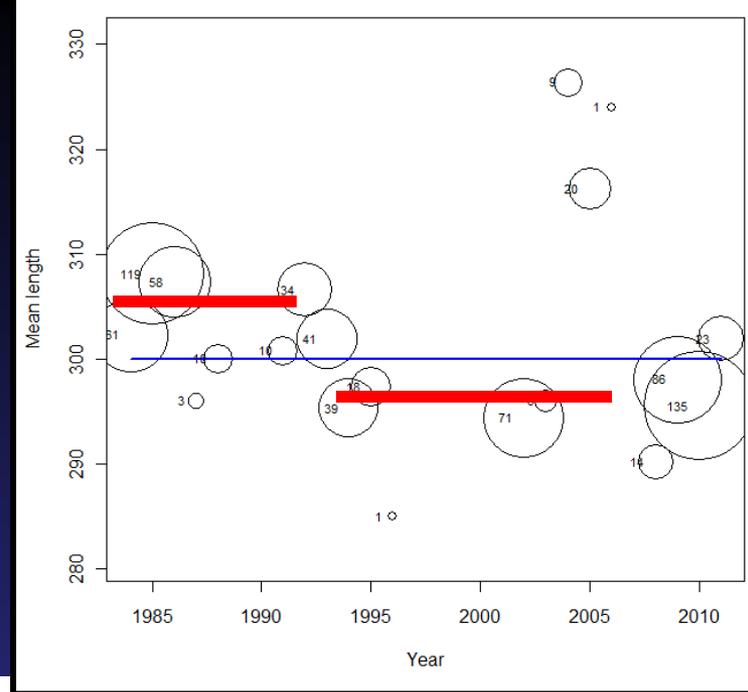
Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by pots and traps in St. Thomas/St. John.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

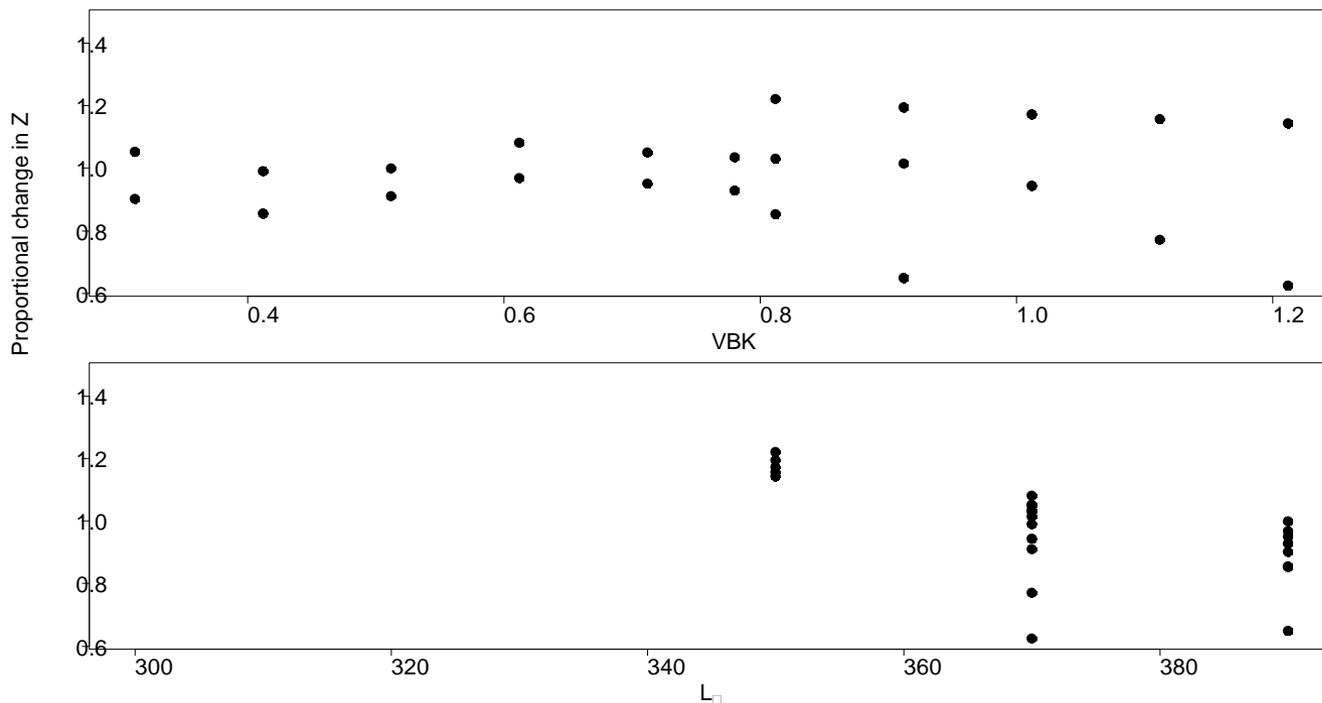


Sensitivity Analysis

- Some runs support change in Z in 1992/1993
- Suggest 60% - 120% increase
- Low Sample sizes



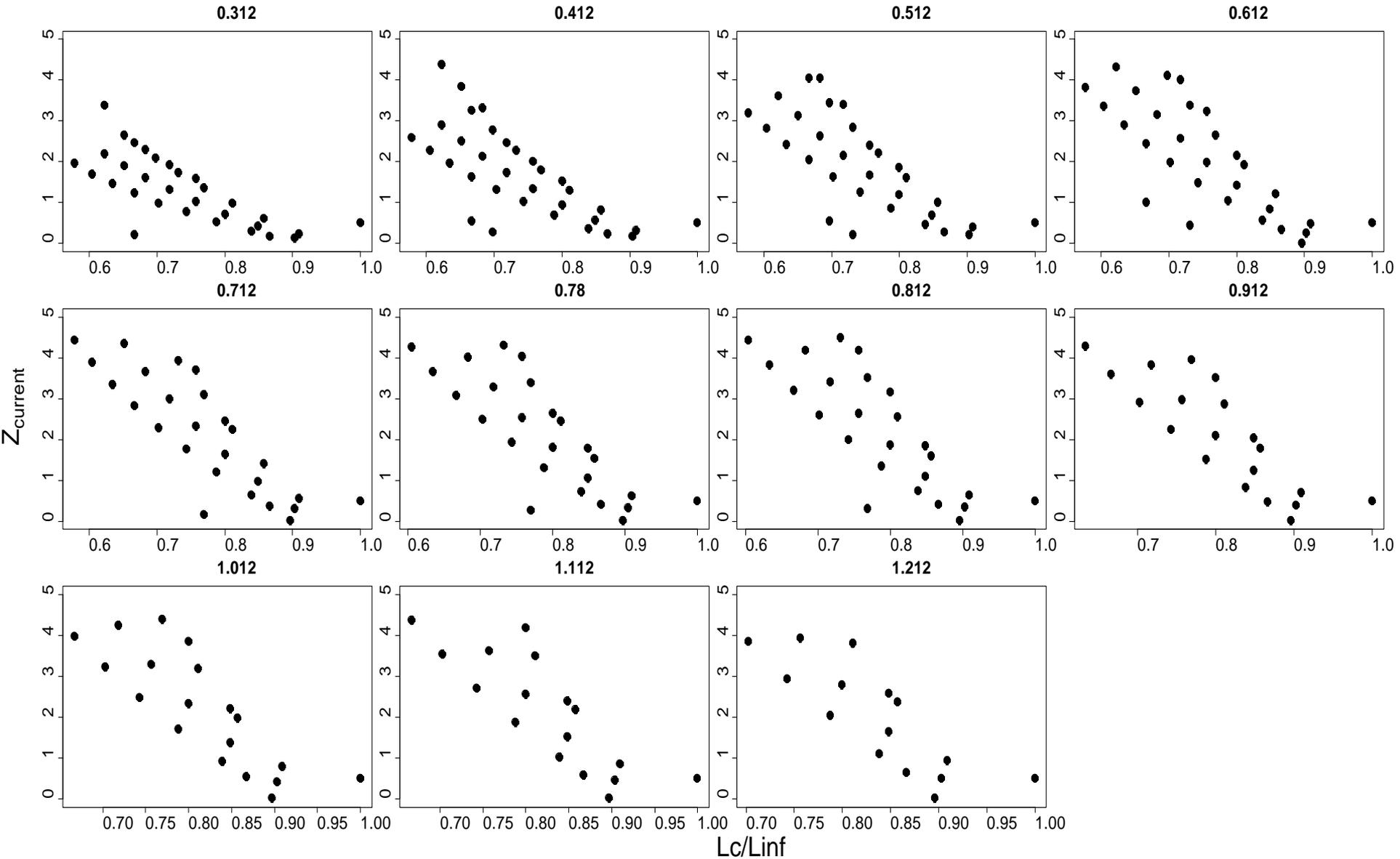
REDTAIL PARROTFISH.POTS AND TRAPS.STTJfinal_summarySens.dat



Proportional change in total mortality (when $\Delta AIC > 5$) as a function of base case L_c (280mm), the full sensitivity range of the von Bertalanffy growth coefficient (K , top panel), and a reduced range of the asymptotic length parameter (L_{∞} , bottom panel).

Sensitivity to Lc/Linf Ratio

REDTAIL PARROTFISH.POTS AND TRAPS.STTJfinal_summarySens.dat

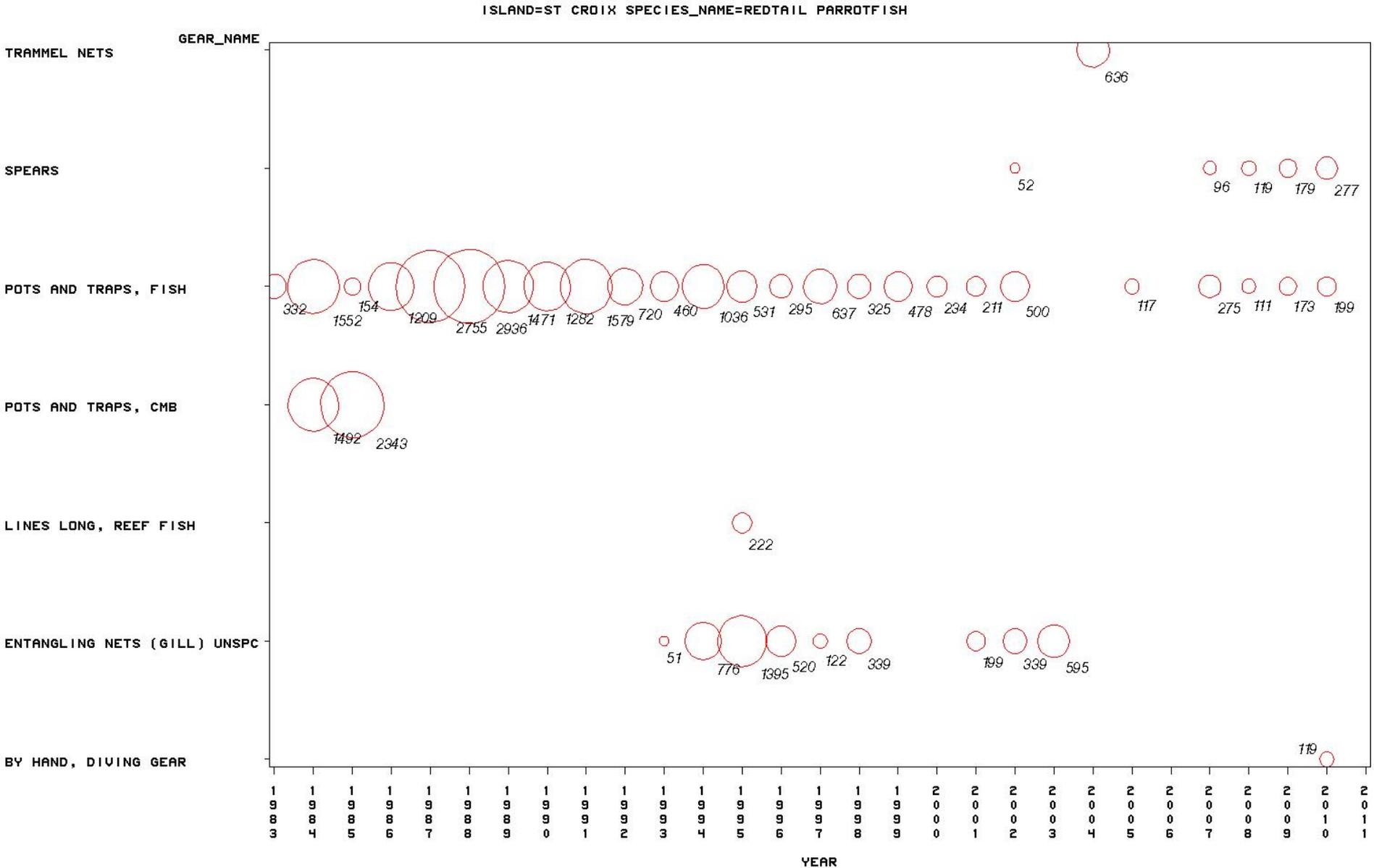


St. Croix

Overview of Data

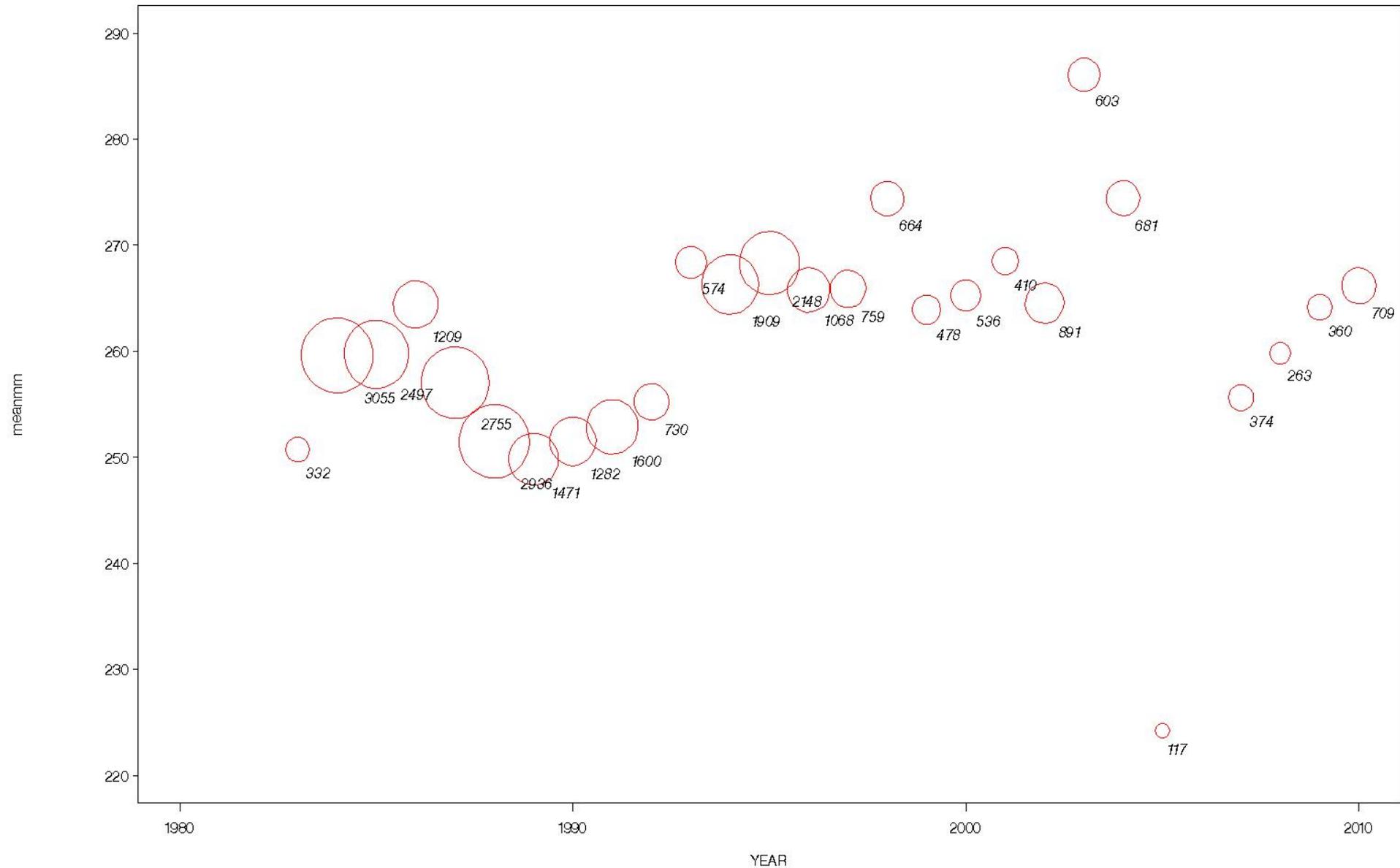
Redtail Parrotfish – St. Croix

Redtail Parrotfish number of sampled fish by specific gear type from St. Croix, 1983-2011.



Redtail parrotfish mean lengths and sample sizes from St. Croix, 1983-2011.

ISLAND= ST CROIX SPECIES_NAME= REDTAIL PARROTFISH

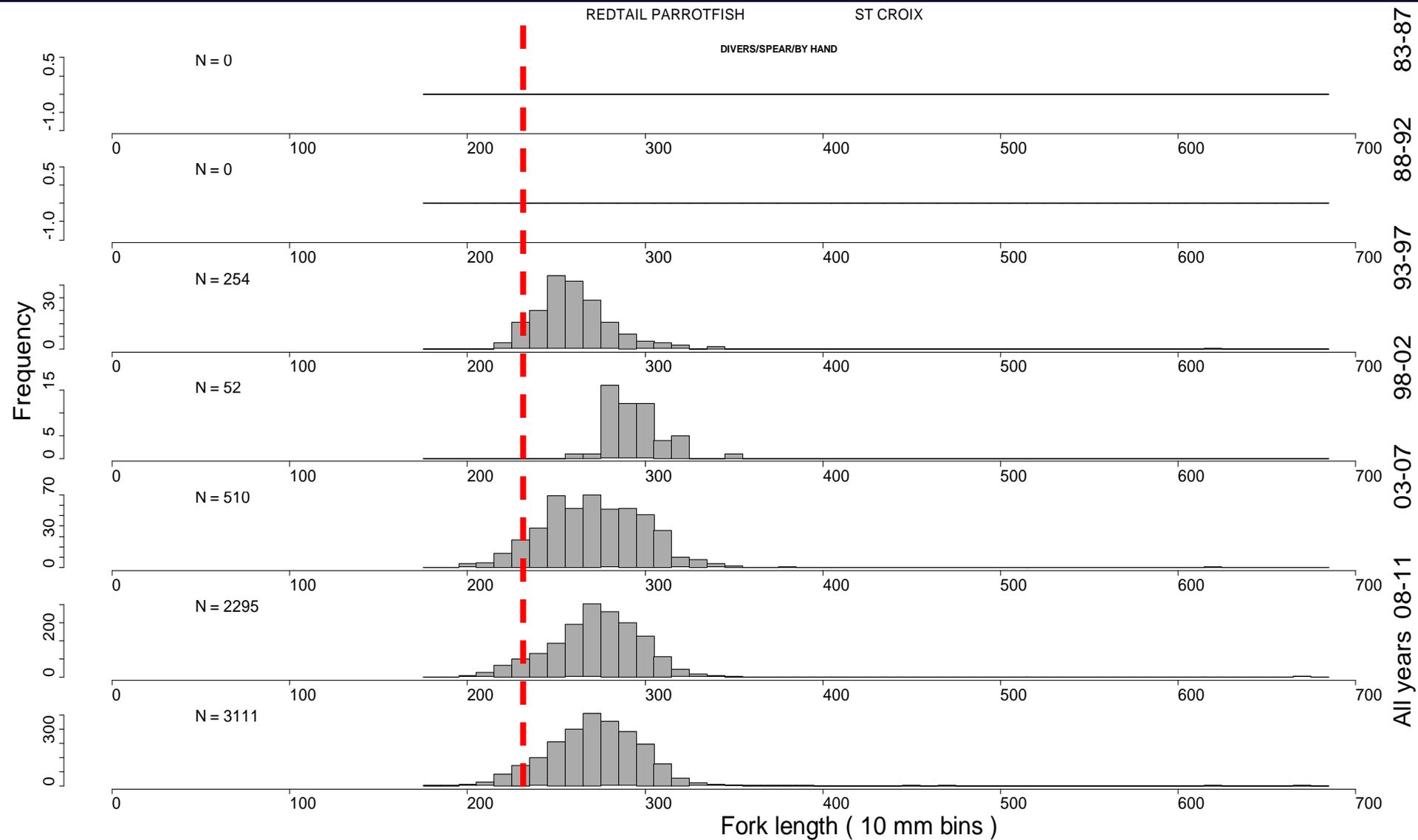


Redtail Parrotfish

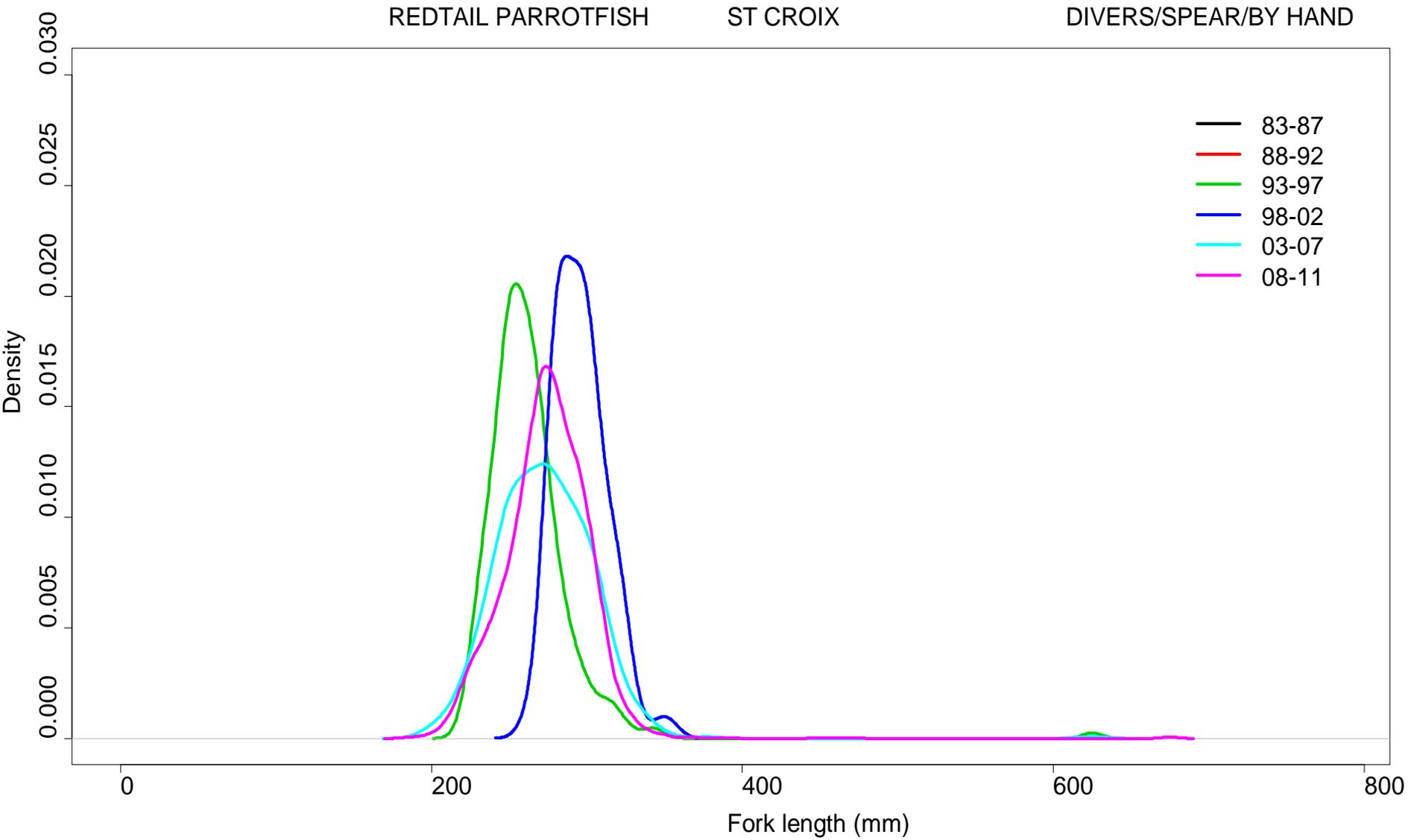
St. Croix

Divers

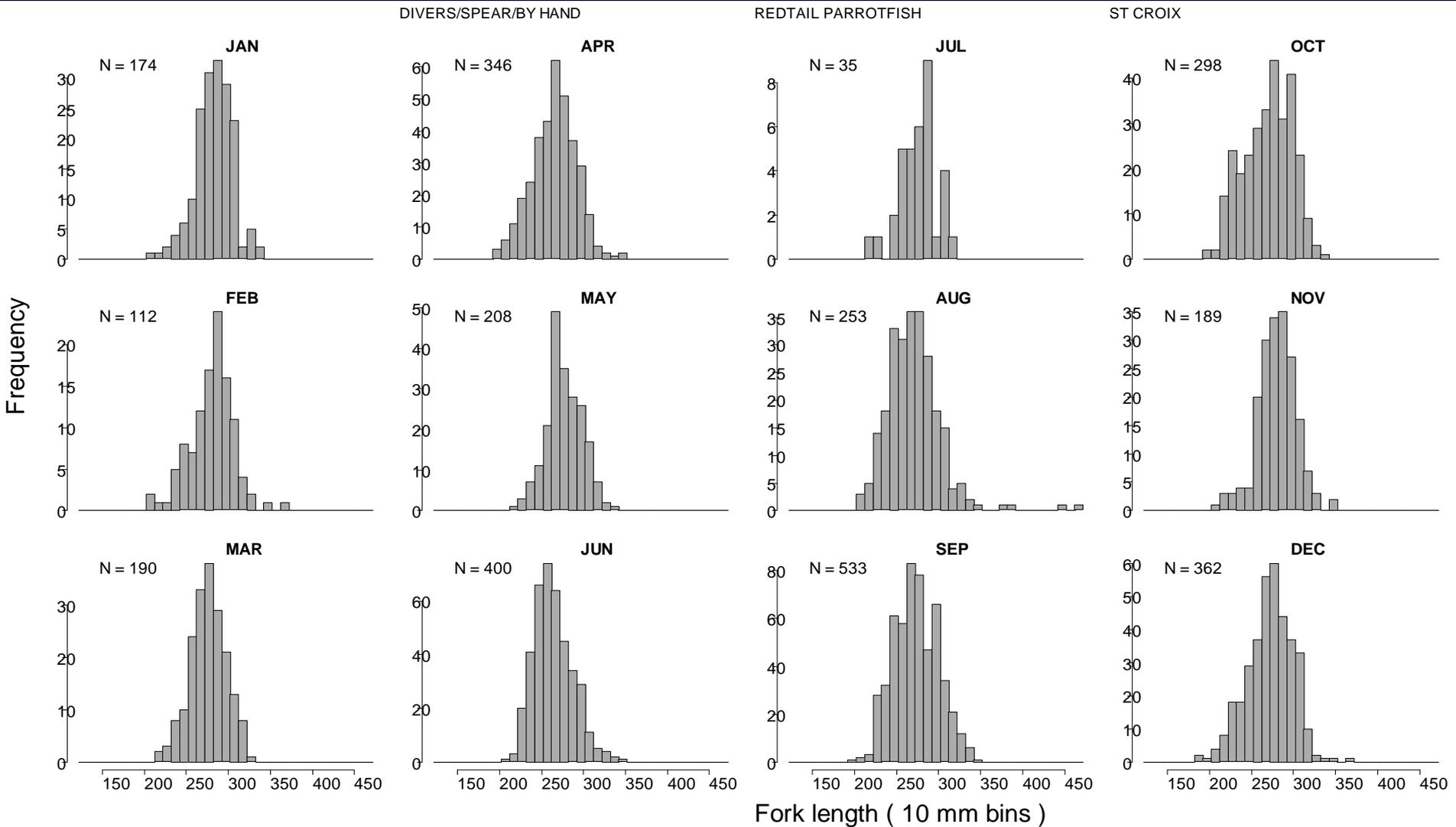
Length frequency distribution for redbtail parrotfish caught by divers in St. Croix.



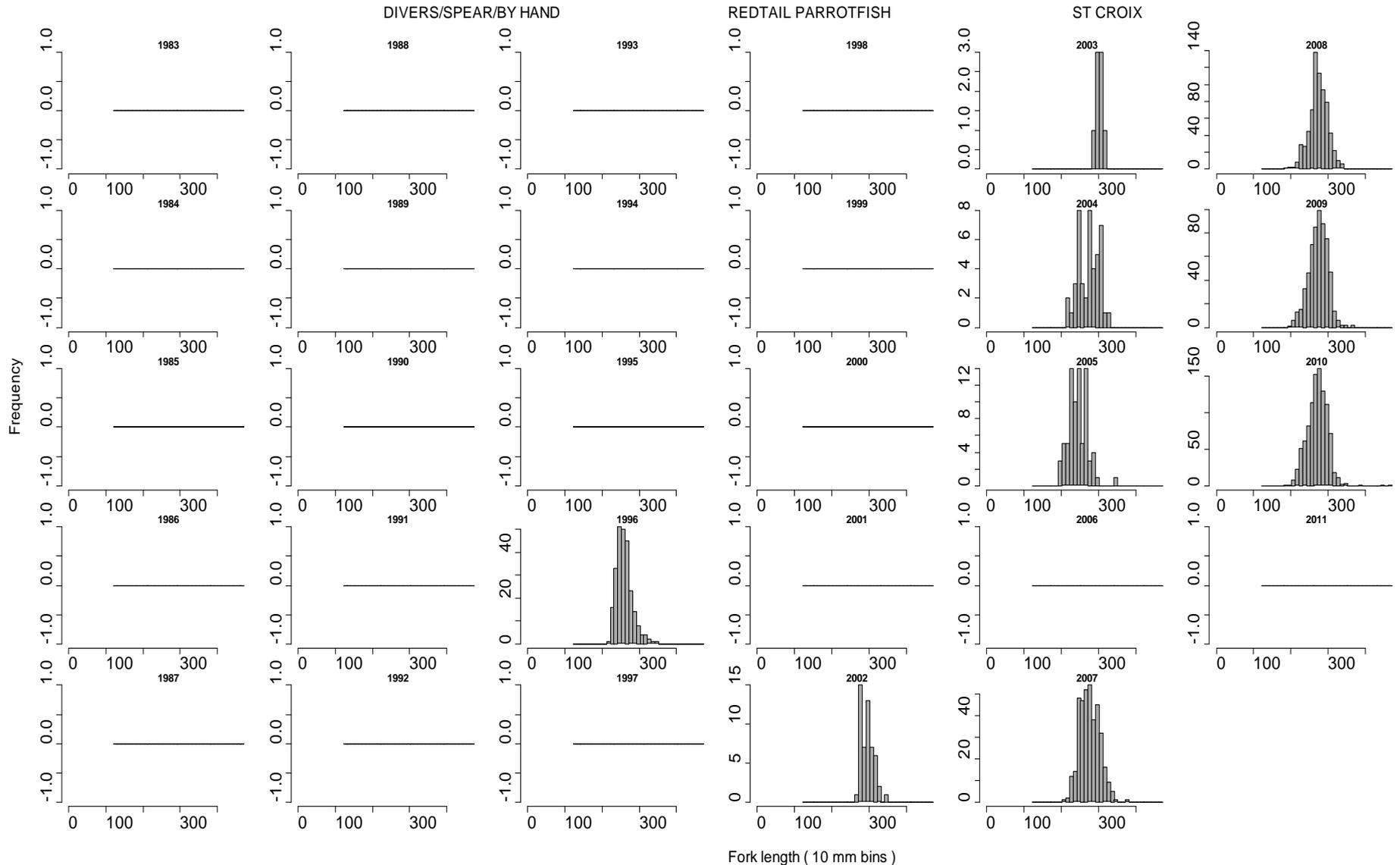
Density plot of observed lengths from the TIP database for redbtail parrotfish caught by divers in St. Croix. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redbtail parrotfish in the diver fishery in St. Croix. N represents the sample size.



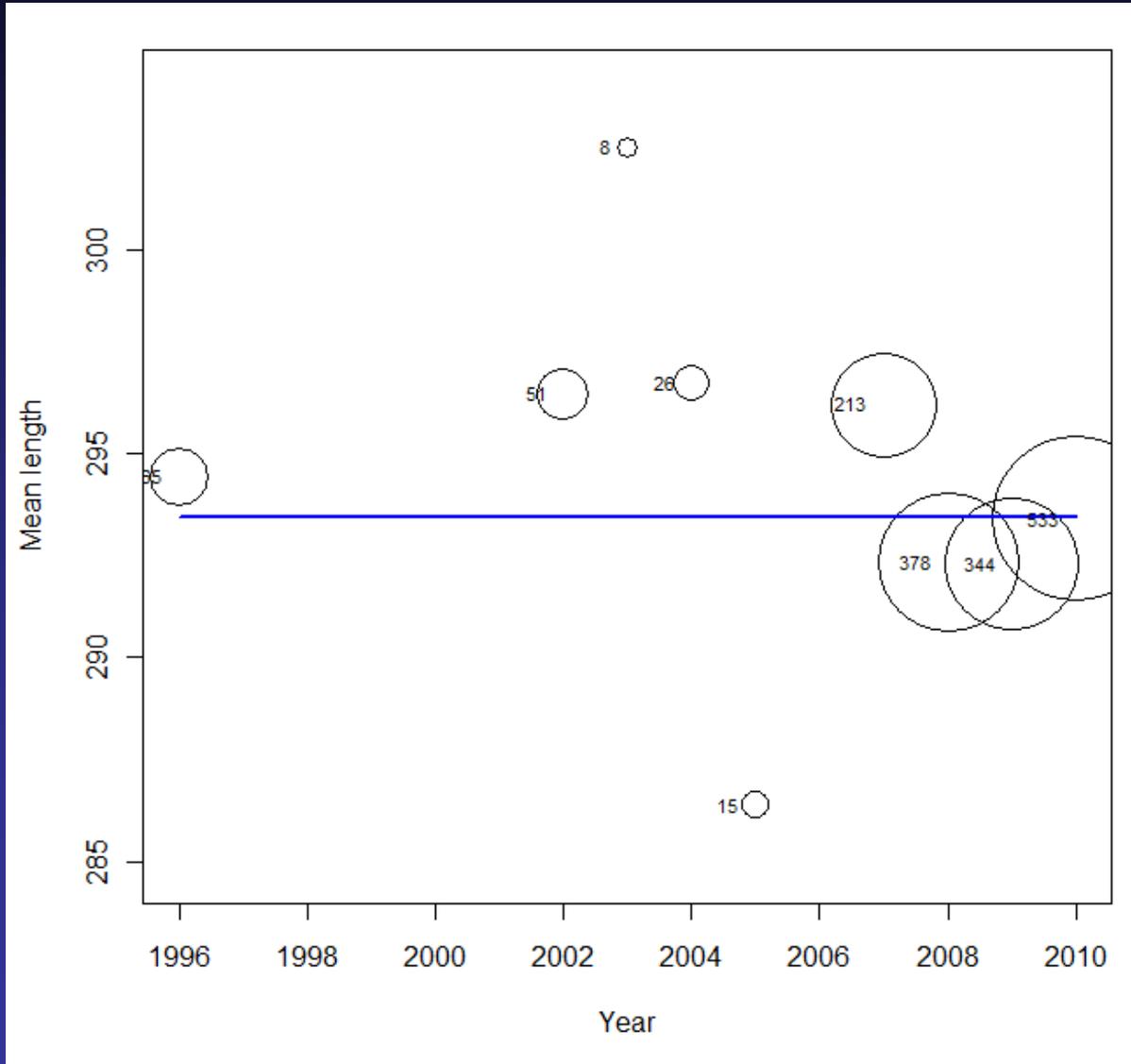
Annual length-frequency histograms for redbtail parrotfish caught by the diver fishery in St. Croix. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



Annual mean length of fully-vulnerable individuals for the base-case scenario for redbail parrotfish caught by divers in St. Croix.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

Base

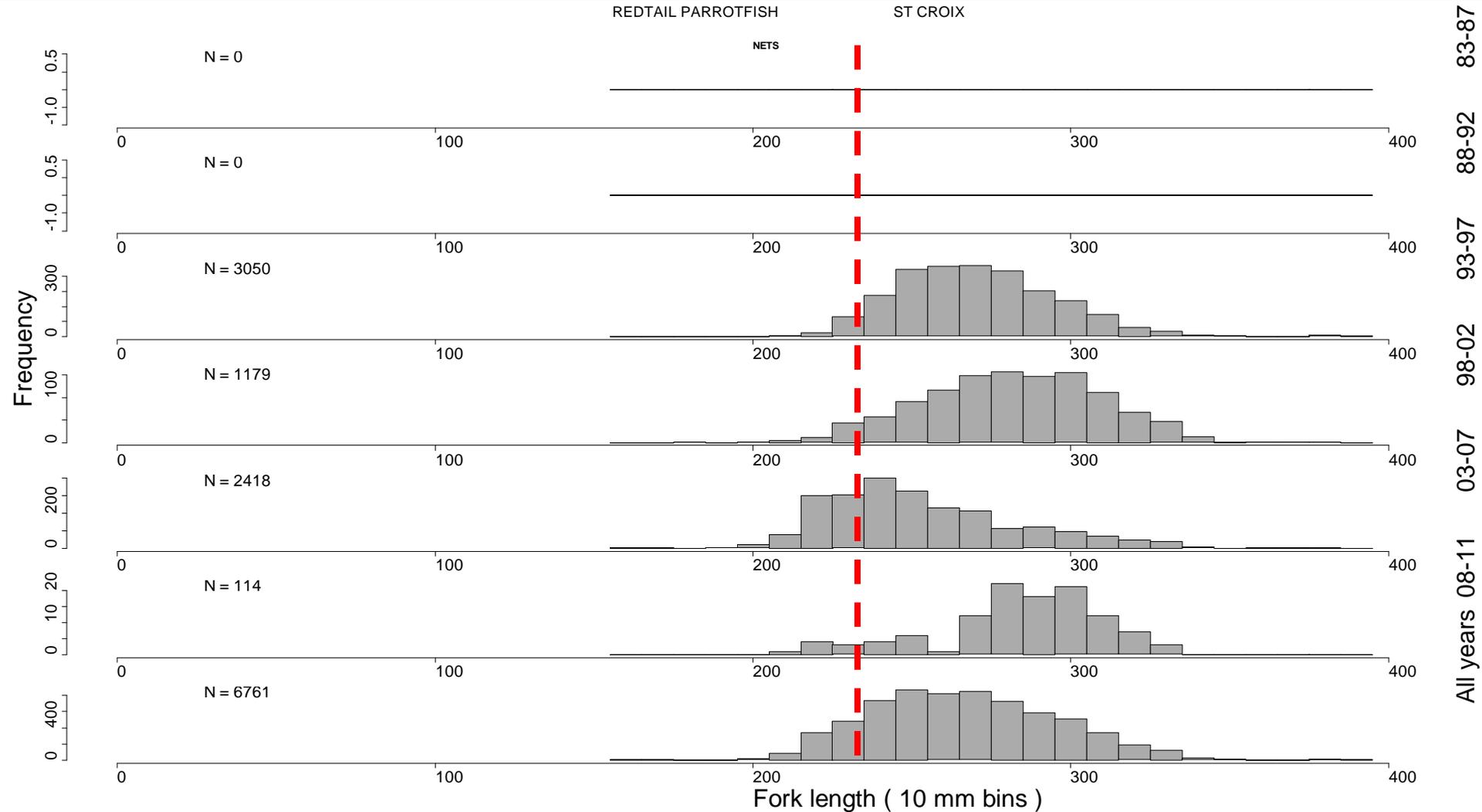


Redtail Parrotfish

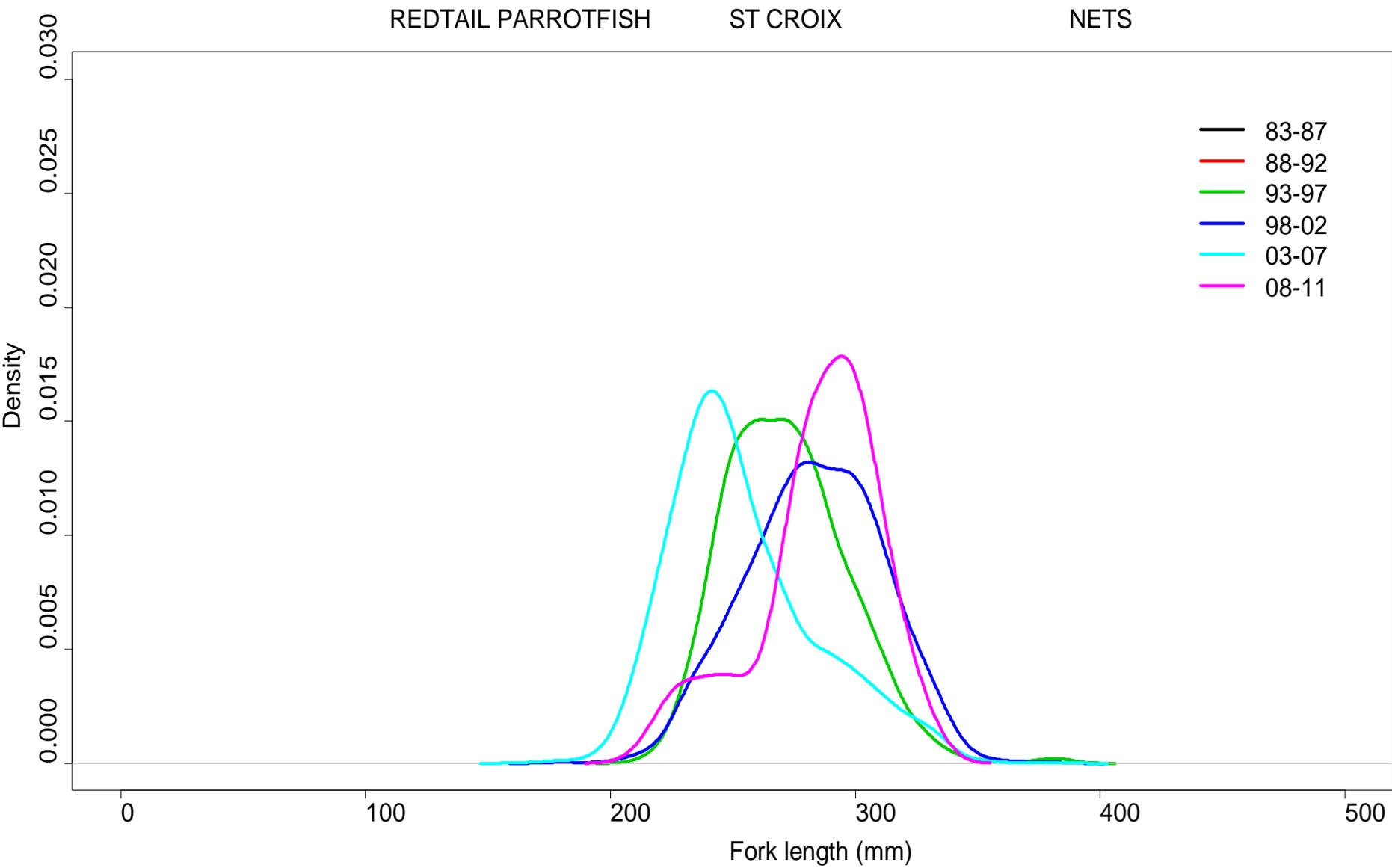
St. Croix

Nets

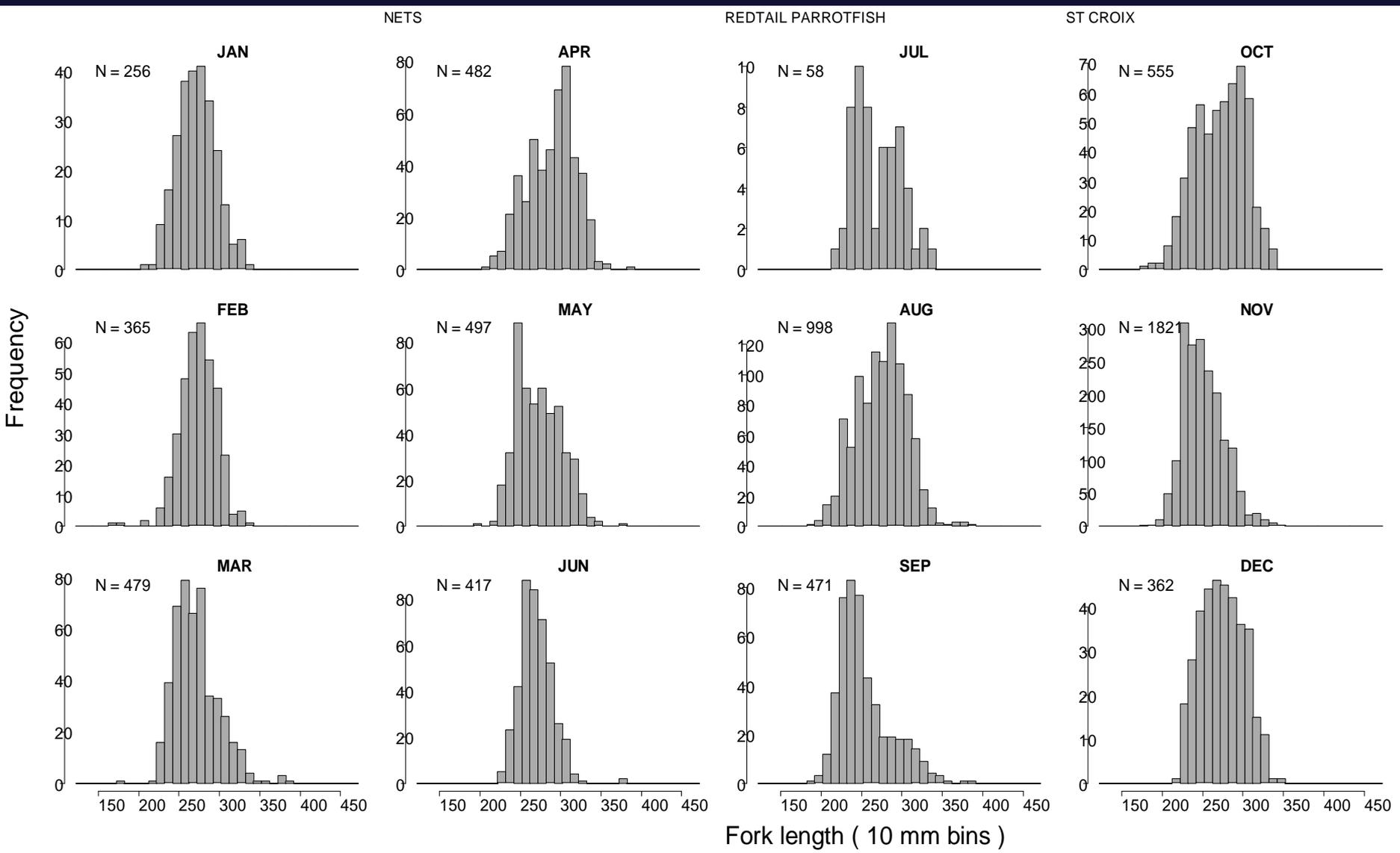
Length frequency distribution for redbtail parrotfish caught by nets in St. Croix.



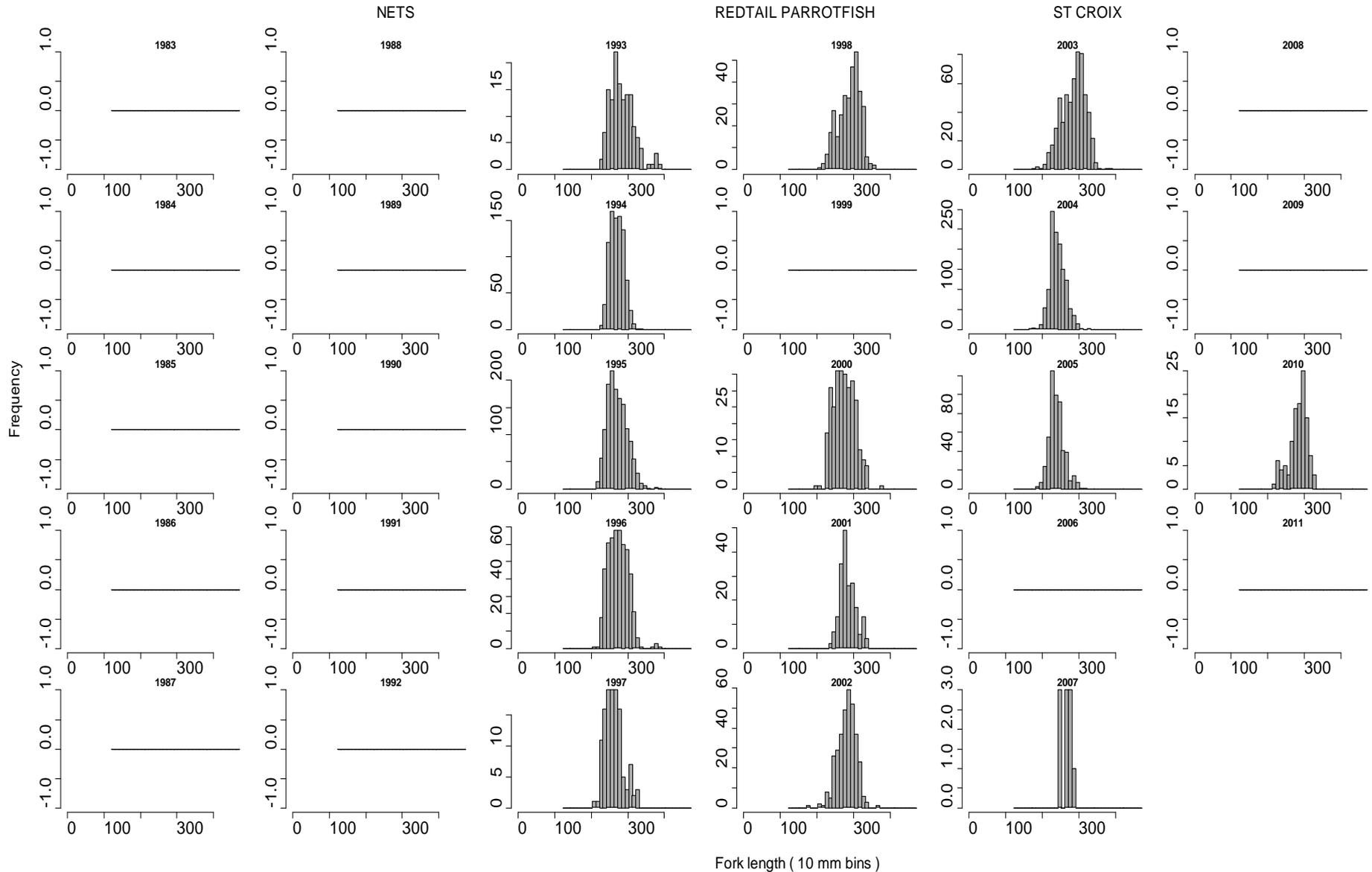
Density plot of observed lengths from the TIP database for redbtail parrotfish caught by nets in St. Croix. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redbtail parrotfish in the net fishery in St. Croix. N represents the sample size.



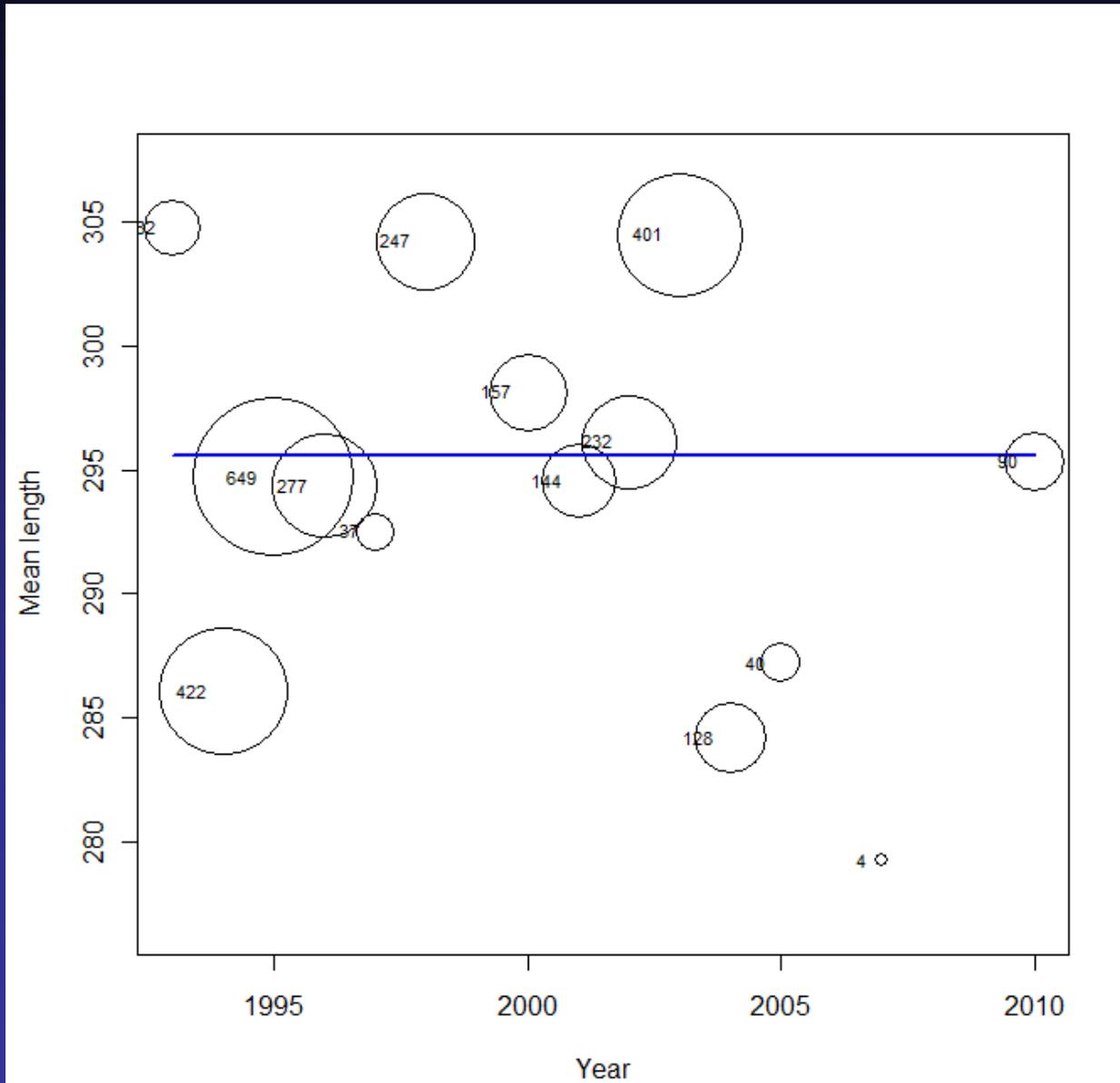
Annual length-frequency histograms for redbtail parrotfish caught by the net fishery in St. Croix. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by nets in St. Croix.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

Base

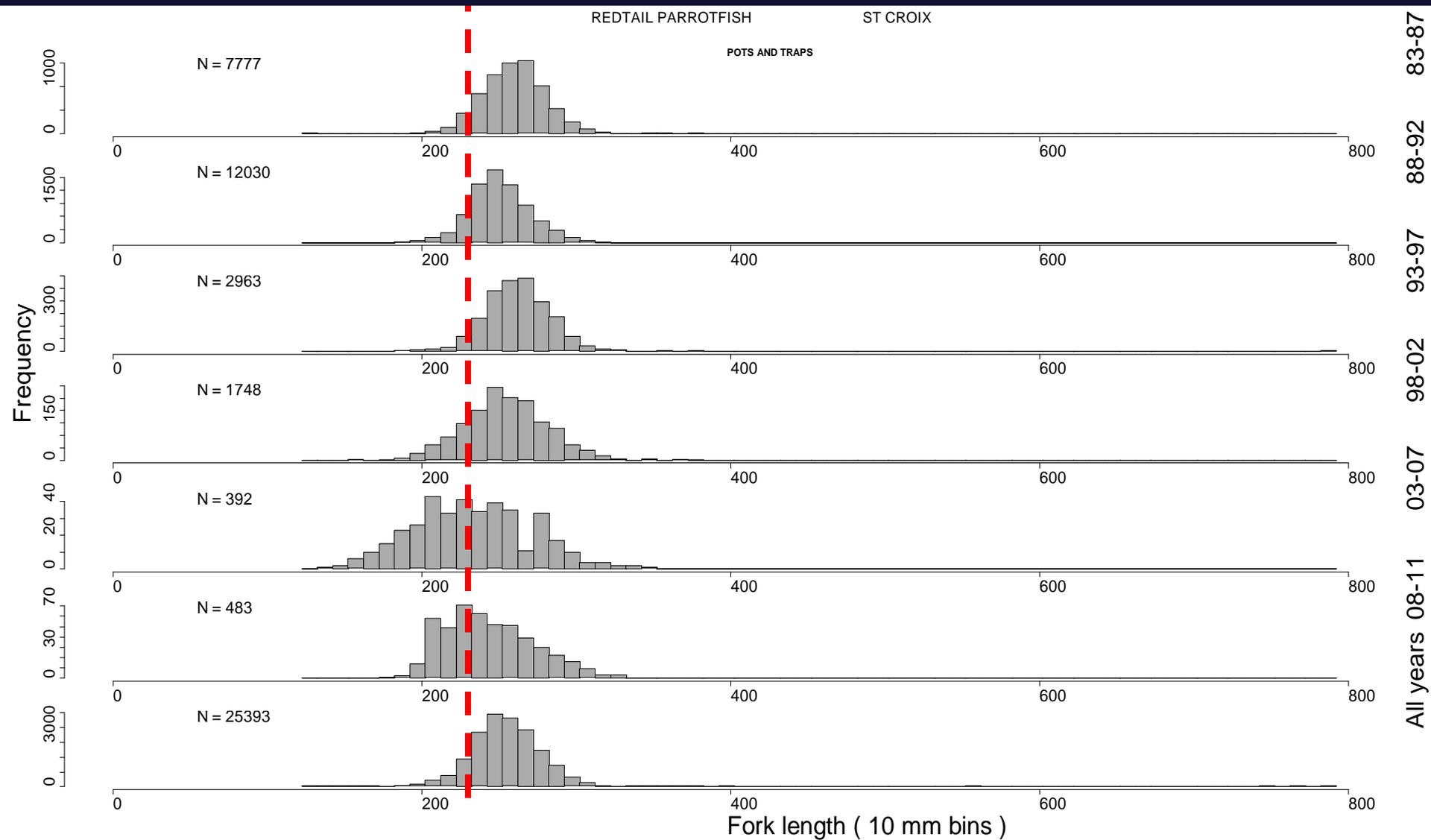


Redtail Parrotfish

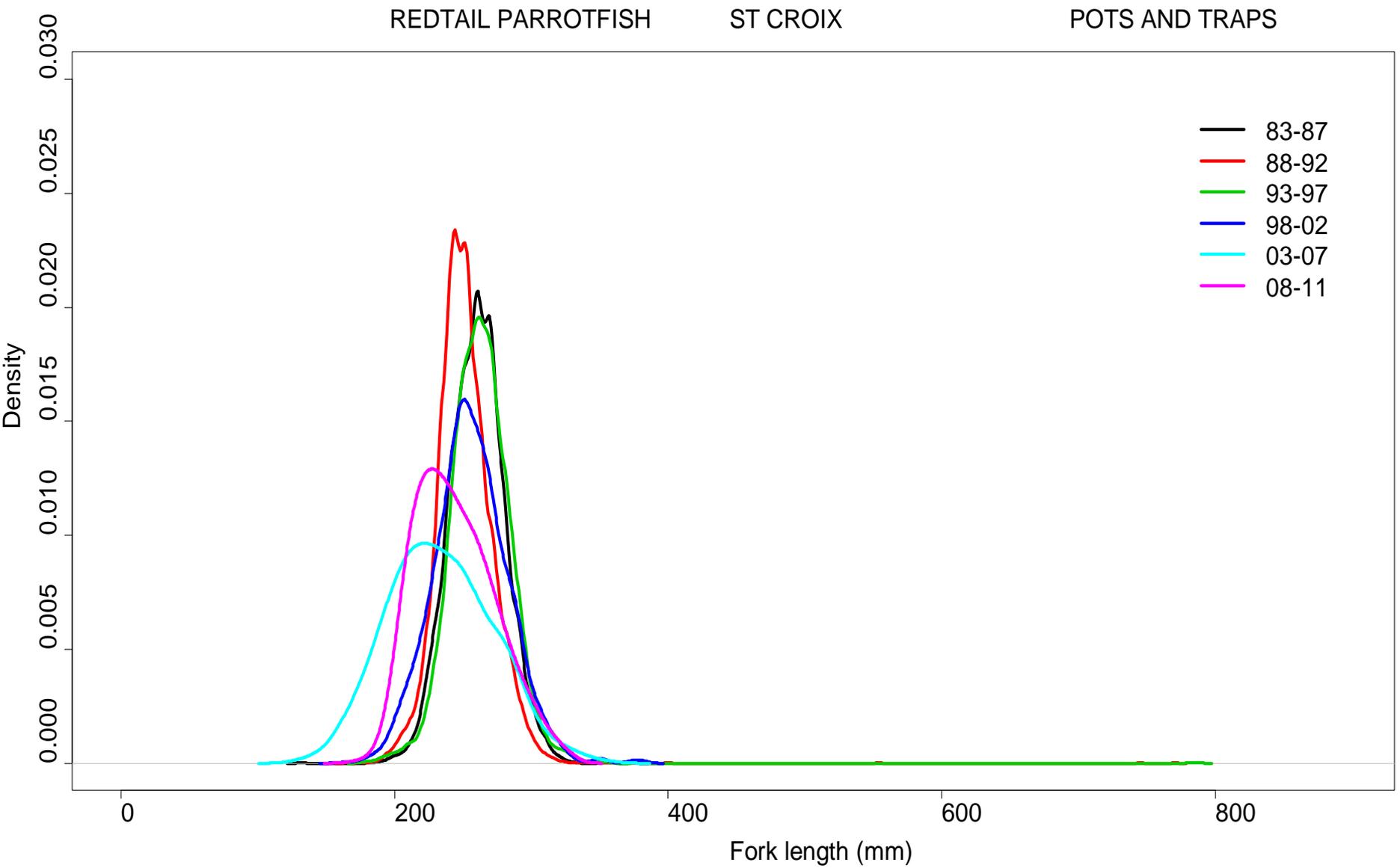
St. Croix

Pots and Traps

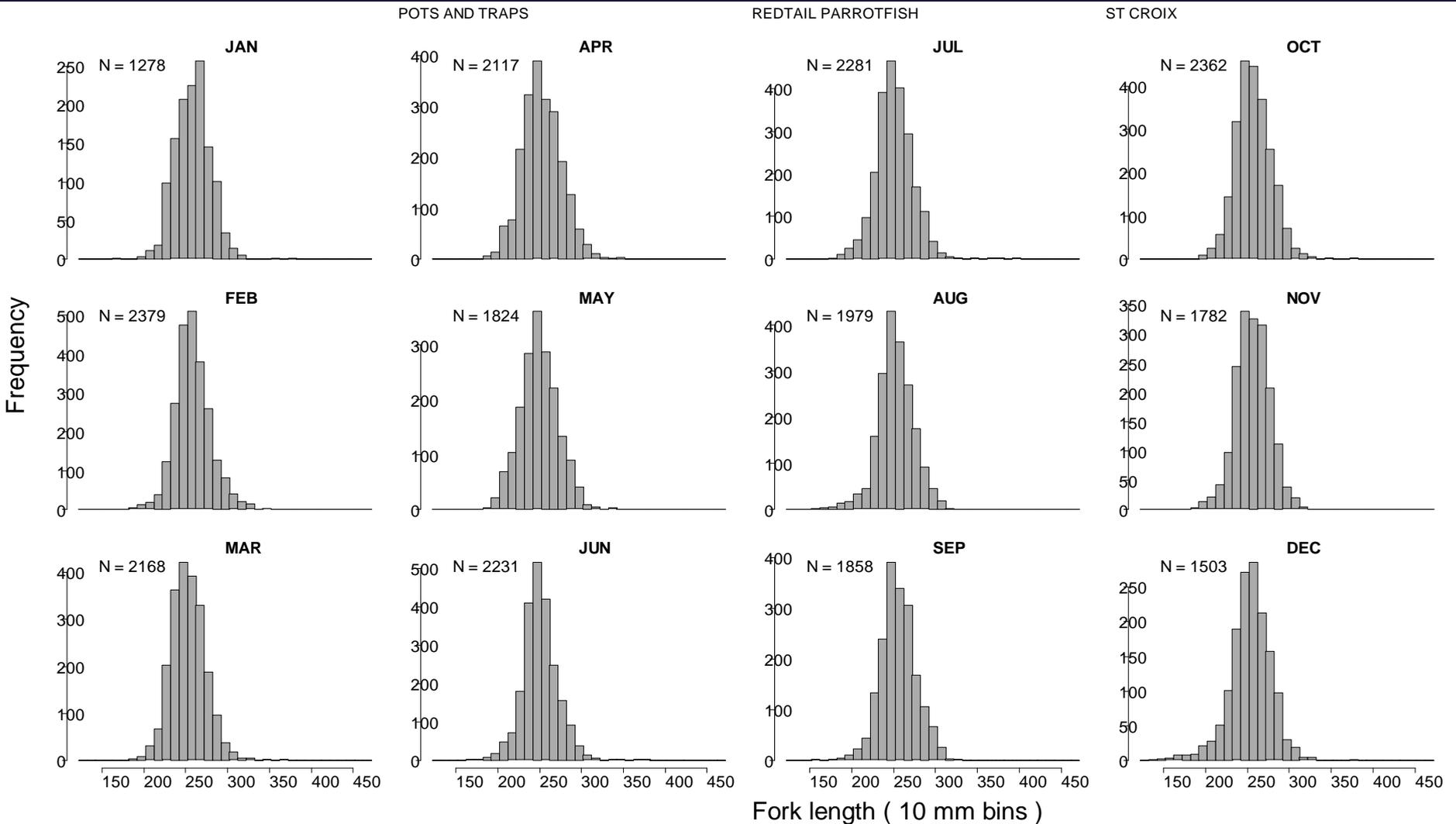
Length frequency distribution for redbtail parrotfish caught by pots and traps in St. Croix.



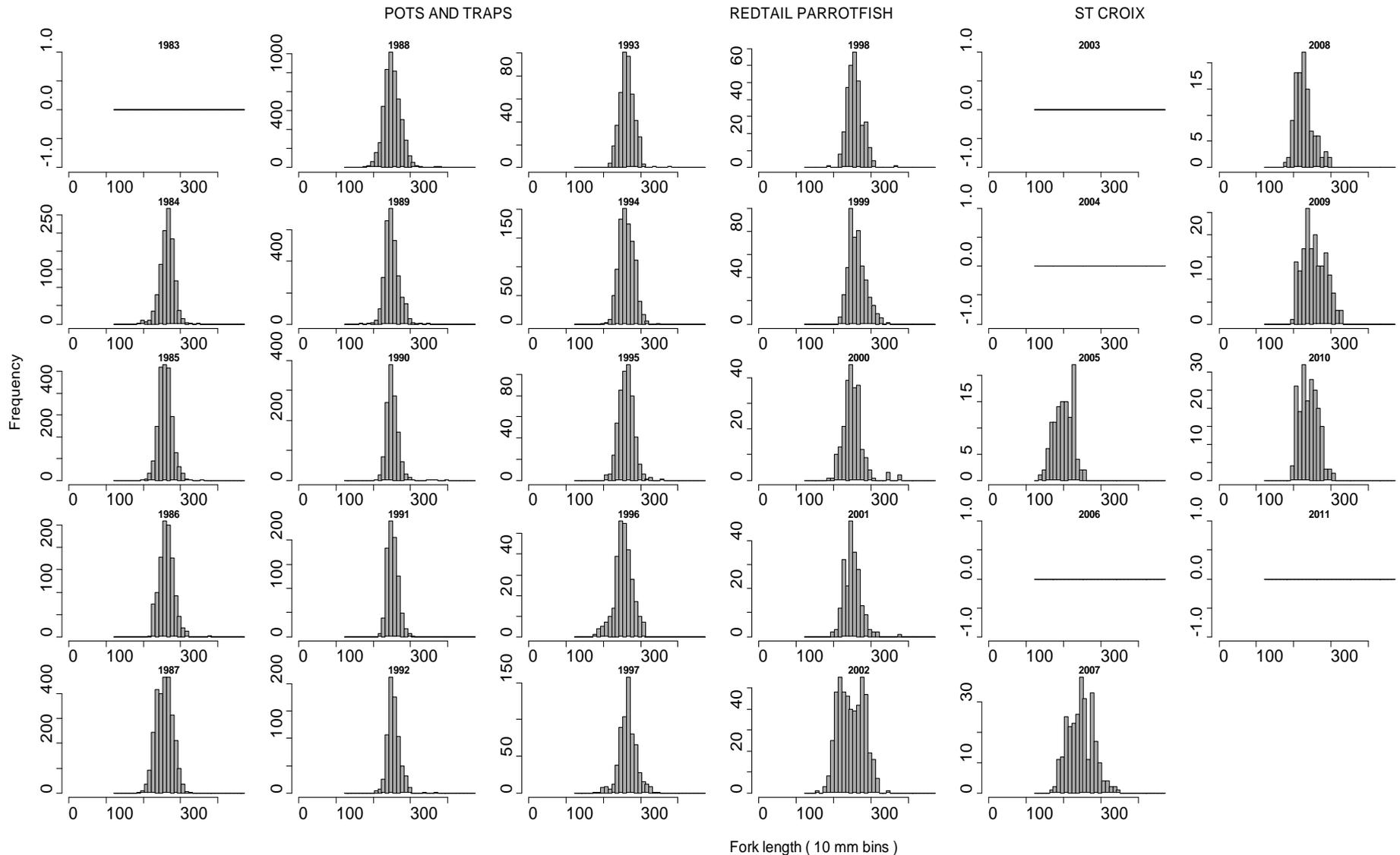
Density plot of observed lengths from the TIP database for redbtail parrotfish caught by pots and traps in St. Croix. Each curve represents a five-year time-period.



Monthly length-frequency histograms, where the length data was aggregated over years, for redtail parrotfish caught in the pot and trap fishery in St. Croix. N represents the sample size.



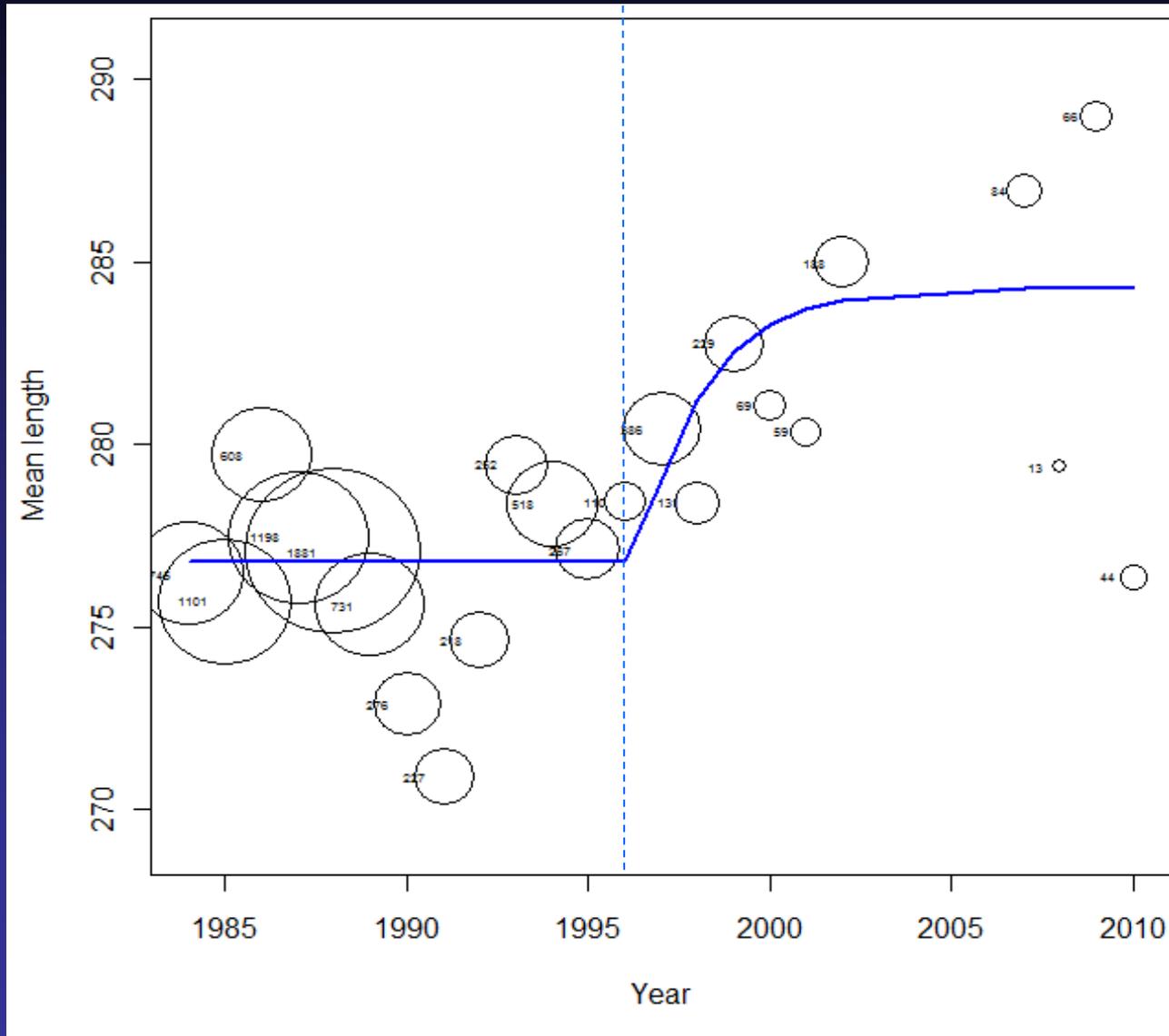
Annual length-frequency histograms for redbtail parrotfish caught by the pot and trap fishery in St. Croix. Flat lines at zero indicate length-data was not collected in those years. Please note that the scale of the y-axis differs for each panel.



Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by pots and traps in St. Croix.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

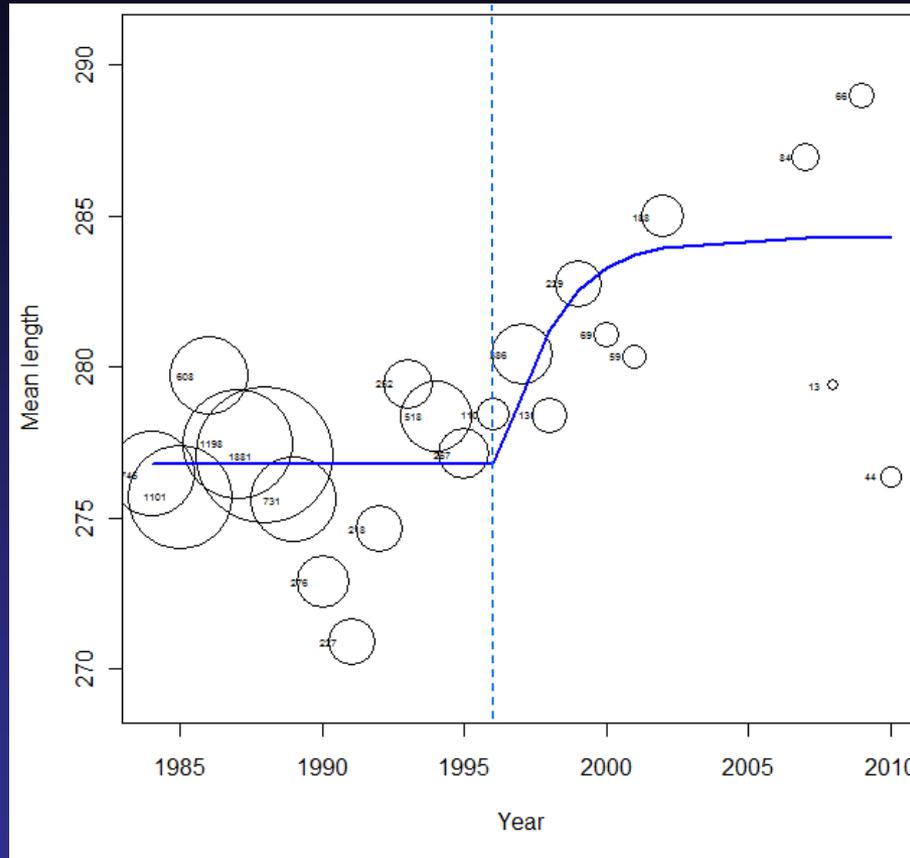
Base



Annual mean length of fully-vulnerable individuals for the base-case scenario for redbtail parrotfish caught by pots and traps in St. Croix.

Bubble size indicates annual sample size relative to other years; the solid blue line represents the line of best fit.

Base

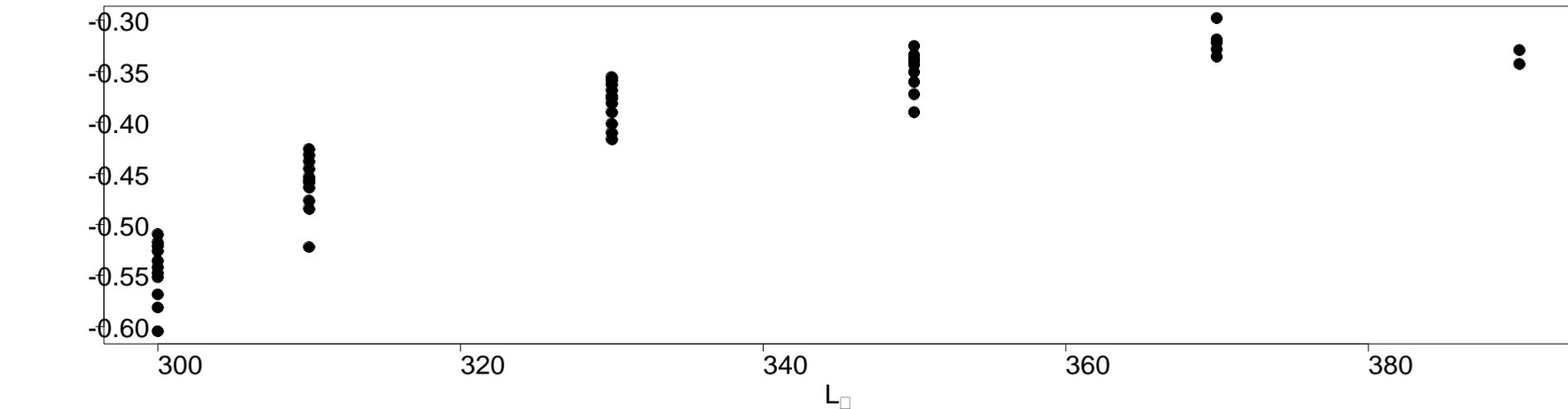
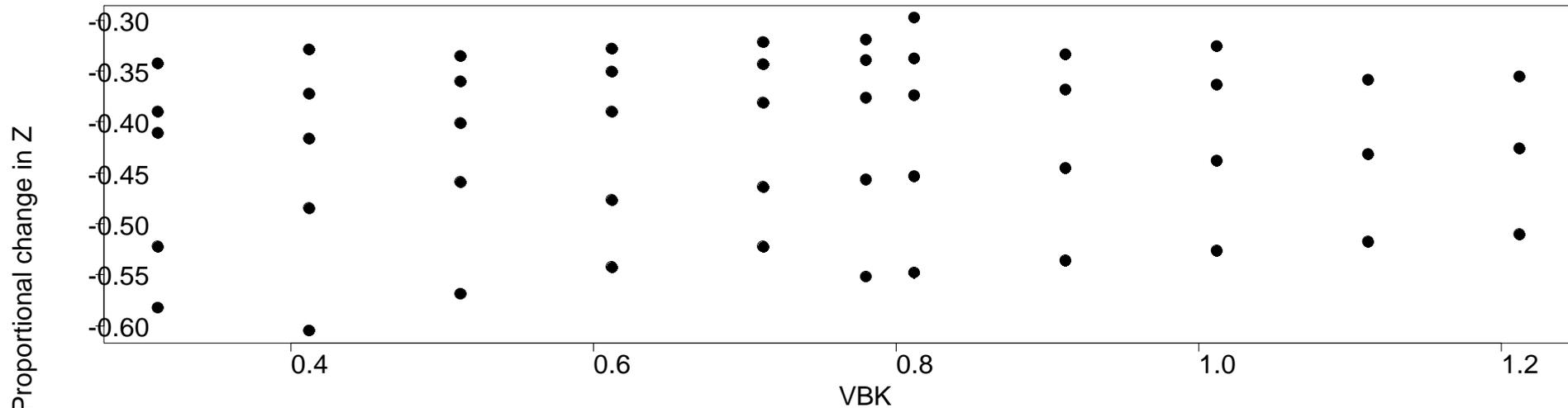


Npar	Nobs	AIC	LLIKE	Z	Sigma	Lc	VBK	L_Inf	Z1	ΔYear1	Z2	ΔYear2	Z3	ΔYear3	Z4
2	23	128.979	62.190	1.138	54.170	262	0.78	300	-	-	-	-	-	-	-
4	23	118.145	53.961	-	37.878	262	0.78	300	1.228	1996	0.550	-	-	-	-
6	23	119.764	51.257	-	33.676	262	0.78	300	1.208	1989	2.590	1991	0.774	-	-
8	23	120.035	46.874	-	27.834	262	0.78	300	1.208	1989	2.505	1991	0.914	1998	0.504

Sensitivity Analysis

- 100% of Sensitivity runs (with reduced range of L_{inf}) strongly support reduction in mortality occurring between 1993 and 1996
- A reduction of between 30% - 60% was predicted in all cases.

REDTAIL PARROTFISH.POTS AND TRAPS.ST CROIX final_summary.dat



Random Walk Extension of SEINE Model

A simple “random walk” structure was added to penalize deviations in the total mortality rate Z from one time period t to the next:

$$Z_{t+1} = Z_t \cdot e^{\delta_t}$$

$$\delta_t \sim N(0, \sigma_Z^2)$$

The subscript t denotes a block of one or more years.

The corresponding negative log-prior is added to the negative log-likelihood to create the penalized objective function (where T is the total number of time blocks).

This negative log-prior is

$$-\log(P(\delta)) = 0.5 \left(T \cdot \log(2\pi) + T \cdot \log(\sigma_Z^2) + \sigma_Z^{-2} \sum_{t=1}^T (\log(Z_{t+1}) - \log(Z_t))^2 \right)$$

